Vehicular Visible Light Positioning with a Single Receiver

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Abstract— Vehicle-to-vehicle (V2V) communication and positioning systems are expected to play an important role in the development of future automated and autonomous vehicle safety concepts. Visible light communication and positioning (VLC and VLP) promise high data rates and cm-level positioning accuracy, respectively, with vehicle head/tail lights. Existing methods for vehicular VLP often require multiple spatially-separated co-operating nodes with either tightly synchronized clocks or precisely known relative locations and they dictate certain modulation schemes or message content for the VLC subsystem. The proposed novel VLP method utilizes a single VLC receiver capable of measuring angle-of-arrival (AoA) on a receiving vehicle (RXV). The method dictates no modulation constraints on the VLC subsystem and no co-operation is required from the transmitting vehicle (TXV) other than disseminating its real-time speed and heading information via VLC. The method uses speed and heading data and two consecutive AoA samples from the same receiver to deduce 2D position of the TXV relative to the RXV with triangulation. Simulation results show the method performs cm-level positioning accuracy at >50Hz rates under realistic road and VLC channel conditions. With such performance, the proposed VLP method enables time-critical traffic safety applications like collision avoidance.

Keywords— autonomous vehicles, visible light communication, visible light positioning

I. INTRODUCTION

Future automated and autonomous vehicle (AV) safety concepts will necessitate that the vehicle takes over road awareness related tasks from the driver and will thus require robust vehicle-to-vehicle (V2V) communication and positioning [1]. Vehicular ad hoc networks (VANET) providing such capabilities are envisioned to improve traffic safety and enable various commercial applications [2].

Dedicated short-range communication (DSRC) around the 5.9 GHz band has been proven for use in VANETs, but DSRC is subject to radio frequency communication (RF) artefacts such as interference, multipath and high deployment cost. Visible light communication (VLC) systems utilize existing LED head/tail lights and low-cost photodiodes for line-of-sight (LoS) communication and are therefore more secure and less susceptible to interference and multipath effects in congested settings. VLC is expected to complement the robust V2V communication architecture of future AVs [3].

Positioning/mapping sensors for AVs range from the LIDAR, RADAR and multi-camera systems which acquire high-detail maps of the environment, to the simpler GPS and inertial sensor-based vehicle positioning systems. Vehicle positioning can also be performed sensor-free at similar performance using communication signals [4], providing a cost-effective and redundant complementary technology. Previous works concerning sensor-free positioning via RF and VLC signals have classified the methods into three main categories: geometric methods, fingerprinting/scene analysis-based methods, and proximity-based methods [5]. These methods utilize the following set of measured “parameters” of

Fig. 1. Visible light communication and positioning system model. θ : angle-of-arrival (AoA). ψ , v , P : vehicle heading, speed and position.

communication signals to deduce positions of nodes in the system: time-of-arrival (ToA), time-difference-of-arrival (TDoA), angle-of-arrival (AoA), phase-difference-of-arrival (PDoA), received-signal-strength (RSS) and message roundtrip-time-of-flight (RTof). Detailed explanations and performance analyses regarding these parameters and the methods which utilize them can be found in [5-6]. While studies on such indoor RF and VLP methods are numerous [6], their extension to the vehicular domain have so far been limited since most methods require either complex co-operation between nodes, precise knowledge of multiple relative node positions (e.g. conventional AoA requires measurement from two nodes for 2D positioning [7]), precise knowledge of channel parameters (e.g. RSS) or tight clock synchronization between nodes (e.g. ToA, RTof). TDoA-based and PDoA-based approaches, which are devoid of such requirements, have been studied for vehicle positioning via utilization of low-cost photodiodes and existing head/tail lights and/or roadside lights [8-9]. AoA-based methods are known to provide higher precision but require complex setups. Although there have also been several AoA-based vehicular VLP approaches with special high-speed VLC-capable cameras [10], these are not low-cost, require extra special-purpose hardware and constrain the VLC subsystem by requiring certain modulation methods to be implemented in order to distinguish multiple nodes.

In this paper, a novel, geometric, AoA-based vehicular VLP method, which utilizes only a single VLC receiver on the receiving vehicle (RXV) to deduce relative 2D positions of transmitting vehicles (TXV), is presented. The method provides high rate relative vehicle position data with cm-level accuracy without requiring co-operation between nodes or prior knowledge of any channel parameters or node locations. The only requirement from the VLC subsystem is that TXV disseminates its speed and heading information. The system model is defined and the AoA-based single-receiver vehicular VLP problem is presented in Section II. The proposed VLP method is presented in Section III. Performance attributes for the VLP method are identified and its performance in realistic road and channel conditions with respect to these attributes are evaluated via simulations in Section IV. After summarizing the formulation and performance results of the proposed VLP method, the paper is concluded in Section V.

II. SYSTEM MODEL AND PROBLEM DEFINITION

A. System Model

The system model considers two vehicles cruising on a flat road, sustaining V2V communication through VLC transceivers mounted on their front and rear. The vehicle to be positioned, TXV, transmits information through this VLC channel to the other vehicle, RXV. In the proposed VLP method, which will be explained in Section III, RXV uses this information and the AoA of the VLC signal for deducing the 2D position of TXV relative to its own frame of reference. The system model is depicted in Fig.1. The assumptions (A#) regarding these two vehicles in the model are listed below:

- A1: TXV and RXV sustain reliable, perfectly synchronized communication by VLC transceivers mounted in their front and rear.
- A2: TXV and RXV measure their real-time speed (v) and global heading (ψ) with on-board sensors.
- A3: TXV includes its real-time speed and global heading information in the VLC transmission to RXV.
- A4: The VLC receivers measure the angle-of-arrival (AoA, θ) of the transmission beam.

Although Fig.1 does not exhaustively depict all cases but only the one where RXV trails TXV, the roles of the two vehicles are interchangeable in this system model as long as the abovementioned assumptions are met.

B. Problem Definition

Conventional AoA-based positioning methods require two distinct angle measurements from spatially separated nodes for 2D positioning with triangulation. Authors in [8-9] have proposed employing two transceivers with known separation on a vehicle for TDoA- and PDoA-based VLP methods respectively. A similar dual-transceiver setup could also be used for an AoA-based method, but accuracy with multi-node setups depends on good calibration, synchronization and accurate knowledge of system parameters such as relative node positions. Hence, a single RX-node AoA-based vehicular VLP method is more favorable.

A single AoA measurement does not define a TXV position, but it constrains the solution set to positions on a line, as shown in Fig.2a. Rather than using two distinct spatially separated AoA measurements from different nodes, it's also possible to use two distinct temporally separated AoA measurements (θ_k) from the same node. This transforms the solution set from single positions to tuples of positions for two time instants, still unbounded, as shown in Fig. 2b. Adding the information of relative distance travelled by the TXV ($d_{TX/RX}$) puts a bound on the solution set of tuples. The tuples which contain positions further apart than the shortest distance between the two lines defined by the two AoA measurements are eliminated (Fig. 2c). For a given TXV relative heading angle (α , different than ψ), there exists a unique tuple within this bounded set in Fig. 2c, which gives the position of TXV relative to the RXV during the two time instants at which the AoA measurements were taken (Fig. 2d). Hence, with a single AoA-measurement-capable VLC receiver on the RXV, finding the relative position of the TXV is possible if the relative heading angle of and distance travelled by TXV, both with respect to the RXV frame of reference, can be inferred from the VLC signal.

Fig. 2. Single receiver VLP problem definition. θ_k : AoA samples at time instant k . $X_{M/N}$: quantity X of M measured in frame of N .

III. PROPOSED VLP METHOD

This section presents the proposed VLP method which runs on the RXV computer and finds the position of TXV with respect to the system model described in Section II. Using:

- RXV and TXV global heading (ψ), measured by on-board sensors, ψ_{TX} transmitted to RXV via VLC
- RXV and TXV speeds (v), measured by on-board sensors, v_{TX} transmitted to RXV via VLC,

the method first deduces:

- TXV relative heading in RXV frame (α),
- TXV relative distance travelled in RXV frame ($d_{TX/RX}$).

RXV then uses these deductions and the AoA of the TXV beam on its receiver to find the relative position of TXV in RXV frame during two consecutive AoA samples (θ_k, θ_{k+1}).

A. Algorithm Details

1) Find Relative Heading of TXV

The relative heading of TXV in RXV frame (α) can be deduced from the global headings (ψ) of and the distance travelled by RXV and TXV ($d_{RX/RX}$ and $d_{TX/TX}$) within the time interval between two consecutive θ measurements (Δt). RXV speed, measured by the RXV sensor, and TXV speed, measured by the TXV sensor and sent over the VLC channel to RXV, are multiplied by Δt to get the respective RXV and TXV distances. The TXV global heading, measured by the TXV sensor, is also included in the same VLC message. Defining $\zeta = \psi_{TX} - \psi_{RX}$, the right triangles shown in Fig. 3a are used to find α , resulting in the following relationship:

$$\alpha = 90^\circ - \text{atan}\left(\frac{d_{TX/TX}(\sin(\tau)) - d_{RX/RX}}{d_{TX/TX}(\cos(\tau))}\right) \quad (1)$$

where $\tau = 90^\circ - \zeta$.

2) Find Relative Distance Travelled by TXV

After finding α , the distance travelled by TXV in RXV frame ($d_{TX/RX}$) needs to be calculated. Applying Pythagorean theorem on the smaller right triangle in Fig. 3a gives $d_{TX/RX}$:

$$d_{TX/RX} = \sqrt{(d_{TX/TX}(\cos(\tau)))^2 + (d_{TX/TX}(\sin(\tau)) - d_{RX/RX})^2} \quad (2)$$

3) Find TXV Relative Position

Finally, with $d_{TX/RX}$ and α known, right triangles shown in Fig. 3b are used for deducing TXV position relative to RXV. With the following definitions:

$$s_\alpha = \sin(\alpha), \quad c_\alpha = \cos(\alpha), \quad \gamma = \tan(\theta_k), \quad \beta = \tan(\theta_{k+1}) \quad (3)$$

the following equation set is obtained from the triangles:

$$\gamma = \frac{y_k}{x_k}, \quad \beta = \frac{y_{k+1}}{x_{k+1}} \quad (4)$$

$$x_{k+1} - x_k = s_\alpha(d_{TX/RX}), \quad y_{k+1} - y_k = c_\alpha(d_{TX/RX}) \quad (5)$$

Solving the set in (4) and (5) for x and y positions results in:

$$x_k = d_{TX/RX} \left(\frac{c_\alpha - \beta s_\alpha}{\beta - \gamma} \right), \quad y_k = \gamma(x_k) \quad (6)$$

Fig. 3. Proposed VLP method. Geometry (a) used for finding α and $d_{TX/RX}$. Geometry (b) used for finding the relative TXV position.

$$x_{k+1} = s_\alpha(d_{TX/RX}) + x_k, \quad y_{k+1} = \beta(x_{k+1}). \quad (7)$$

where x_k, x_{k+1}, y_k and y_{k+1} are x and y positions of TXV in RXV frame for consecutive time instants k and k+1.

B. Performance Attributes

The performance of the proposed VLP method is dependent on the following attributes (PA#):

- PA1: Accuracy of the VLP method inputs: θ, ψ and v measurements. While these metrics depend on sensor/hardware specifications for all three, θ is additionally subject to signal-to-noise ratio (SNR) on the additive white gaussian noise (AWGN) VLC channel since it's inferred from the received waveform
- PA2: The relative magnitudes of vehicle dynamics ($\Delta\alpha/\Delta t$ and $\Delta v/\Delta t$), sensor measurement rates, VLC message rate and VLP positioning rate.

The method finds the exact relative TXV position if it receives exact ψ, v and θ input data (perfect PA1) and the relative vehicle dynamics is constant for the two consecutive time instants k and k+1 (perfect PA2, i.e. $\Delta\alpha/\Delta t$ and $\Delta v/\Delta t$ are 0). Non-idealities in realistic road and channel conditions and

practical constraints on the system cause deteriorations in PA_s , and thus degradation of VLP performance. While the ψ and ν sensor inaccuracies and SNR-dependent θ measurement inaccuracy constitutes PA_1 -related degradations, PA_2 deteriorates as the following condition weakens:

$$\min(\text{sensing rate}, \text{VLC rate}) \geq \text{VLP rate} \gg \left(\frac{\Delta\alpha}{\Delta t}, \frac{\Delta\nu}{\Delta t} \right) \quad (8)$$

Since the VLP method running on the RXV requires real-time ψ and ν , the limit for maximum VLP rate is the minimum of the ψ and ν sensing rate and the VLC link data rate (over which the RXV receives TXV ψ and ν). On the lower side of (8), since the VLP method assumes constant relative vehicle dynamics over the two consecutive θ samples on which it operates, VLP performance degrades as the difference between the VLP rate and the vehicle dynamics gets narrower for non-constant vehicle dynamics. For constant vehicle dynamics, there is no hard limit on the minimum VLP rate.

IV. PERFORMANCE EVALUATION

The goal of the simulations is to demonstrate the performance of the proposed VLP method under practical system constraints and realistic road and VLC channel conditions by introducing realistic deteriorations to the PA_s defined in Section III. Each deterioration is applied in turn by tuning the others to their realistic minimums to maintain the clarity of presentation.

A. Simulation Setup

An end-to-end vehicular VLC system simulator (i.e. from the TXV LED to the RXV receiver) was built in MATLAB® for generating the RX VLC signal, from which the proposed VLP method infers the only input information it needs: θ , ψ and ν . The simulator considers an automotive grade standard 1000 lumen power LED (typical for taillights and low-intensity headlights) as TX [11-12]. The LED intensity is modulated with binary frequency shift keying (BFSK) for simplicity, and the VLC link rate is chosen as 25 kbps, larger than the requirement for collision avoidance in a 2005 report by the Vehicle Safety Communications Consortium (VSCC) [13], which is approximately 5 kbps [14].

The AoA-capable RX, shown in Fig. 4, is based on a low-cost quadrant photodiode (QPD) such as [15]. It contains a spherical convex lens for focusing the rays from the TX LED into a spot which traverses the QPD cells with respect to the AoA (marked θ on Fig.4) [16]. AC-coupled amplifiers on each QPD cell reject ambient low frequency noise and feed the signal to the ADCs on the RXV microcontroller. The readings from the four cells carry the same TX VLC signal with different power, which is used for deducing AoA [17-18]. Combination of the four cells is used for VLC. While various alternative AoA-capable RX were evaluated in literature for both VLP and MIMO VLC [18-20], the QPD is the simplest fast sampling VLC RX with AoA measurement capability.

B. Simulation Results

1) PA_1 – Effect of SNR on θ and VLP

The TXV movement with respect to RXV causes the intensity reaching RXV to change, resulting in a dynamic SNR. Therefore, while evaluating VLP performance sensitivity to θ by changing SNR, the “worst-case SNR” over a trajectory is considered as the signal integrity metric.

Fig. 5 demonstrates the effect of VLC SNR on θ and thus on VLP performance (PA_1) for a straight and steady TXV trajectory relative to RXV (i.e. $\Delta\alpha/\Delta t$, $\Delta\nu/\Delta t = 0$, therefore no

Fig. 4. AoA-capable VLC receiver. Dimension dH represents active area. dL, dF and dX: lens diameter, focal length and distance from QPD.

effect from PA_2) and perfect ψ and ν sensor measurements. TXV is slightly faster than RXV which is cruising at 36 km/h, moving away from it in an oblique direction ($\alpha = 16.7^\circ$) for 2 seconds. Fig. 5c is drawn in RXV frame, therefore RXV movement is not depicted. Performance becomes worse towards the end of the trajectory, where SNR is lower. In order to combat noise in the proposed VLP method, the ADC samples used for θ measurement can be averaged over time. This action decreases θ sample rate and thus VLP rate since it needs an increased buffer time to average out the zero-mean AWGN component, but it increases positioning accuracy and stability. While the upper VLP rate limit is 500Hz due to the sensing rate as stated in (8), there is no practical lower limit since vehicle dynamics are constant throughout this straight trajectory. The general lower limit for VLP rate can be regarded as the safety application requirement, which is ≥ 10 Hz for cooperative collision avoidance [14]. Fig.6 summarizes effects of VLC SNR on VLP performance, showing that acceptable performance can be achieved even for very low worst-case SNR levels and very high VLP rates ($>> 50$ Hz) when the accuracy-rate trade-off is well utilized.

2) PA_1 – Effects of ψ and ν Sensor Inaccuracies on VLP

A large set of feasible TXV “mini-trajectories” within crash proximity of the RXV (i.e. ~ 10 m radius within the field-of-view of the RX) are tested in this simulation. For each mini-trajectory, which consists only of two consecutive relative TXV positions over Δt , the VLP error for a typical 50 Hz VLP rate and for different ψ and ν accuracy values are calculated, and average error over all mini-trajectories are saved. Mini-trajectories where $\Delta\theta/\Delta t \approx 0$ are excluded from this simulation since the term $(\beta - \gamma)$ in the denominator of (6) approaches 0 when $\Delta\theta/\Delta t \rightarrow 0$, rendering the method unstable. As stated before, the proposed VLP method requires two distinct consecutive θ measurements to function properly.

Fig. 7 demonstrates the effects of ψ and ν sensor accuracy on VLP performance (PA_1) for all mini-trajectories, assuming perfect θ . Commercially available sensors provide accuracy levels of ~ 0.3 km/h for speed and up to 0.02° - 0.1° for heading [21] at ≥ 500 Hz rates. The lower data point in Fig. 7 shows that for the best realizable ψ and ν accuracy (0.02° and ~ 0.3 km/h respectively), the method provides cm-level accuracy at 50 Hz rate. Since ψ error is dominant and it multiplies with the distance travelled between consecutive samples, higher VLP rates decrease error. Hence, higher VLP rates would enable the use of even lower grade sensors for the same performance.

3) PA_2 – Sensing, VLC, VLP Rates vs. Vehicle Dynamics

Fig. 8 and Fig. 9 demonstrate the effects of PA_2 on VLP performance for a partially curved trajectory where

Fig. 5. *PAI*, assessment of θ -SNR sensitivity. TXV travels trajectory shown in (c) in 2 seconds ($\alpha = 16.7^\circ$). Worst case SNR 20dB and VLP rate 50Hz in (a). Worst case SNR 2dB and VLP rate 10Hz in (b). Since the proposed VLP method infers the Y position result from X, their error performance is similar. Thus, only the time plot of X is shown. Mean error is ~ 11 cm (orange line) under both the typical (a) and the extra noisy (b) channel conditions.

Fig. 6. *PAI*, summary of the simulation in Fig. 5. Performance over different VLP rate and worst case SNR values are shown. The lower two data points belong to the results shown in Fig. 5.

$(\Delta\alpha/\Delta t, \Delta v/\Delta t) = 0$ does not hold for all consecutive θ sample pairs on the trajectory. Worst-case SNR is set to 40dB and ψ and v are assumed perfect to realistically minimize effects of *PAI* and focus only on *PA2*, and a relatively fast VLP rate of 100 Hz is chosen for clearer demonstration of the condition in (8). While the method performs well during Part I and III where vehicle dynamics are slower than VLP rate, the method starts to lose stability in Part II when the $\gg$ condition in (8) starts to narrow for consecutive θ samples (i.e. $\Delta\alpha/\Delta t$ starts to

Fig. 7. *PAI*, assessment of VLP sensitivity to ψ and v accuracy. Data points are average VLP errors over all feasible mini-trajectories for each ψ and v accuracy level.

increase), as shown in the middle part of the trajectory. Therefore, for faster vehicle dynamics, a higher VLP rate needs to be chosen while sacrificing noise mitigation. Additionally, the last part (IV) of the trajectory is arranged so that $\Delta\theta/\Delta t \approx 0$ to show that the method becomes unstable under that condition.

A comparison of the simulated performance of the proposed VLP method with existing state-of-the-art vehicular positioning methods is given in Table I.

V. CONCLUSION

This paper presents a novel VLP method that finds the 2D position of a transmitting vehicle (TXV) relative to a receiving vehicle (RXV) utilizing a single AoA-capable VLC receiver. Instead of two AoA measurements from spatially separated nodes, the method uses two consecutive AoA measurements from the same node and heading and speed information for triangulation. No co-operation is required from the TXV other than disseminating its real-time heading and speed information via VLC. Key performance attributes for the method were identified and discussed. Simulations testing the key performance attributes show that the method can achieve cm-level accuracy at $>50\text{Hz}$ rates for moderate SNR levels (20dB) in realistic road scenarios and similar accuracy at even very low SNR ($\sim 2\text{dB}$) by averaging more samples and lowering VLP rate. Accuracy levels of commercially available heading and speed sensors were found adequate for attaining cm-level VLP accuracy. Performance suffers for VLP rates that are closer to vehicle dynamics such as in curved trajectories since the method assumes relative TXV heading and speed stays constant throughout the two samples. In summary, the proposed VLP method provides relative 2D vehicle position data at cm-level accuracy and high rates with commercially available components, which enables its use as a complementary and redundant technology in time-critical traffic safety applications such as collision avoidance.

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TABLE I. COMPARISON WITH OTHER POSITIONING METHODS

Method	Accuracy (cm)	Rate (Hz)
Proposed vehicular VLP method	$\sim 1\text{--}10$	$\gg 50$
Existing vehicular VLP methods [4, 8, 9]	≥ 10	≥ 50
Sensors, on average (RADAR/LIDAR ^a /Vision) [22]	≥ 10	≤ 50

^a RADAR/LIDAR fails to provide sufficient across-track (lateral) accuracy

Fig. 8. Simulation for the deterioration of PA_2 trajectory

Fig. 9. Simulation for the deterioration of PA_2 VLP error

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