Minimum Length Scheduling for Power Constrained Harvest-then-Transmit Communication Networks

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Abstract—We consider a wireless powered, harvest-then-transmit communication network, which consists of a single antenna, energy and information access point (AP) and multiple, single antenna, batteryless users with energy harvesting capabilities. At the beginning of a time frame, the AP broadcasts energy in the downlink to the users. Then, users transmit their data to the AP in the uplink, using their harvested energy. We formulate the optimization problem with the objective of minimizing the total schedule length, subject to the constraints on the minimum amount of data to be sent to the AP, and unlike previous studies, the maximum transmit power for the information transmission. This problem is nonlinear and non-convex. The solution is based on bi-level optimization, consisting of optimizing the transmit power allocation of the nodes for a given energy harvesting time and searching over harvesting time allocation. We also propose a heuristic algorithm in which we incorporate the optimal solution of a single user network. Simulation results demonstrate that under appropriate network conditions, our proposed algorithms provide close-to-optimal results with a reasonable run time compared to another time minimization algorithm which does not integrate the uplink power constraint.

I. INTRODUCTION

5G enables massive machine-to-machine (M2M) communication with a total of 50 billion connected devices by 2020 [1]. A drawback for M2M communication is that devices are usually powered by batteries with limited capacity. Renewing batteries may not be feasible due to the operating conditions of a network (e.g., body implants, toxic environment, large scale networks). A promising, environment independent and fully controllable solution is radio frequency (RF) energy harvesting (EH), in which energy is harvested from the propagating electromagnetic waves. Hence, RF-EH is suitable for M2M applications with critical QoS and delay constraints, such as healthcare, automotive, security etc.

The joint design of downlink (DL) energy transfer and uplink (UL) information transmission (IT) in wireless powered communication networks (WPCNs) has been investigated for throughput maximization [2]–[5]. Harvest-then-transmit (HTT) network with multiple users under half [2] and full [3] duplex AP, the utilization of information and energy beamformings for multiple antenna networks [4], [5] are studied. [6] imposes a maximum UL power limit on users while maximizing energy

efficiency. However, none of these works utilized the EH and IT lengths to meet the delay requirements.

While increasing EH and IT lengths has a positive effect in the throughput performance, there exists a trade-off between the harvested energy, throughput, and delay [2]. In the context of tradeoff, when to switch between EH and IT [7], the coupling effect between EH of a user and ITs of the previously assigned users [8], the coupling between consecutively scheduled users in a chain model [9], and a half-duplex HTT network [10] are studied for total time minimization. Although the transmit power of a device is restricted by its hardware in practice, aforementioned works allowed the users to transmit at any power. To the best of our knowledge, there is no work in the literature recognizing the UL transmit power limit while minimizing the total transmission time.

In this paper, we consider a HTT network with multiple EH users and a half-duplex AP, deploying TDMA. We jointly optimize DL and UL time and UL power allocations to minimize the total scheduling length of the network. The problem is constrained by the data amounts that each user has to send in the UL and the maximum UL transmit power. This is a nonlinear and non-convex problem and we propose a logarithmic time algorithm that provides the optimal solution within an error bound following the derivations of the optimality conditions. And, we propose a suboptimal algorithm, employing the optimal solution we derive for a single user network.

The paper is organized as follows: Section II describes the system model and the assumptions used throughout the paper. Section III formulates the minimum length scheduling problem. Section IV provides the near optimal solution with the associated algorithm. The suboptimal algorithm is given in Section V. Section VI presents simulations and performance evaluation. Concluding remarks are given in Section VII.

II. SYSTEM MODEL AND ASSUMPTIONS

- We considered a HTT network with K users, U_i , $i = 1, \dots, K$, and one AP, all are equipped with single antenna and operating over the same frequency.
- In a transmission frame, AP broadcasts a DL signal to the users for the first τ_0 amount of time for EH, while τ_i amount of time is assigned for the IT from U_i to the AP, later. The users equip neither any other energy source nor

battery to store the harvested energy for latter use; thus, all the EH and ITs take place within a frame.

- The AP maintains the synchronization of the network and informs the users about the current schedule at the beginning of each time frame. This signaling is assumed to be fixed length and negligible.
- The channel power gains from the AP to U_i in the DL and from U_i to the AP in the UL are denoted by h_i and g_i , respectively. The channels are block-fading and gains are perfectly known at the AP.
- U_i has to send a certain amount of data in a frame, denoted as D_i , which is constant and non-negative.
- AP transmits with constant power P_A in the DL, which is sufficiently large such that the energy harvested due to the receiver noise is negligible. Then, the total energy harvested by U_i is $E_i = \zeta_i P_A h_i \tau_0$, where $\zeta_i \in (0, 1)$ is the EH efficiency of U_i , and assumed to be constant in this paper.
- Each U_i uses a portion of its harvested energy, η_i , in the UL, $0 < \eta_i \leq 1$. Continuous power model is assumed and the average transmit power of U_i in the UL for IT, P_i , is given by

$$P_i = \eta_i \frac{E_i}{\tau_i} = \frac{\eta_i \zeta_i P_A h_i \tau_0}{\tau_i}. \quad (1)$$

- There is an upper limit on the power radiated by U_i , mostly due to the hardware limitations of the power amplifier, denoted as P_i^{max} .
- There is no interference during IT. The instantaneous UL transmission rate for U_i is

$$R_i(\tau, \eta_i) = W \log_2 \left(1 + \frac{g_i \eta_i \zeta_i P_A h_i \tau_0}{\tau_i \sigma^2} \right), \quad (2)$$

where $\tau = [\tau_0 \tau_1 \dots \tau_K]$, W is the channel bandwidth and σ^2 is the power of Additive White Gaussian Noise (AWGN) at the AP.

III. OPTIMIZATION PROBLEM

The optimization of transmit power and time allocation to minimize the scheduling length of a HTT WPCN with one AP and K users, subject to the constraints that each user has a certain amount of data to send to the AP with a maximum allowable transmit power is mathematically formulated.

The goal of the optimization problem is to minimize the sum of the EH and IT lengths. (3b) represents the data requirement that U_i has to meet in each frame. (3c) states that the transmit power of U_i cannot exceed a maximum allowable power. (3d) defines the range of η_i , serving to adjust U_i 's UL transmit power. (3e) is the non-negativity constraint for the time allocation variables.

$$\min_{\tau, \eta} \quad \sum_{i=0}^K \tau_i \quad (3a)$$

$$\text{s.t.} \quad \tau_i W \log_2 \left(1 + \frac{g_i \eta_i \zeta_i P_A h_i \tau_0}{\tau_i \sigma^2} \right) \geq D_i, \quad \forall i = 1, \dots, K, \quad (3b)$$

$$\frac{\eta_i \zeta_i P_A h_i \tau_0}{\tau_i} \leq P_i^{max}, \quad \forall i = 1, \dots, K, \quad (3c)$$

$$0 < \eta_i \leq 1, \quad \forall i = 1, \dots, K, \quad (3d)$$

$$\tau_i \geq 0, \quad \forall i = 0, \dots, K. \quad (3e)$$

Lemma 1. P_i , defined in (1) is a nonlinear and neither convex nor concave function of (τ, η_i) .

Proof. Let $\nabla^2 P_i(\tau, \eta_i)$ denote the Hessian of $P_i(\tau, \eta_i)$ defined in (1). Then, the eigenvalues of $\nabla^2 P_i(\tau, \eta_i)$, denoted as $\lambda_{\nabla^2 P_i}$, are

$$\lambda_{\nabla^2 P_i} = \begin{bmatrix} 0 & \\ \frac{A_i((\tau_0^2 + \tau_i^2)(\eta_i^2 + \tau_i^2))^{\frac{1}{2}} + A_i \eta_i \tau_0}{\tau_i^3} & \\ -\frac{A_i((\tau_0^2 + \tau_i^2)(\eta_i^2 + \tau_i^2))^{\frac{1}{2}} + A_i \eta_i \tau_0}{\tau_i^3} & 0 \end{bmatrix} \quad (4)$$

where $A_i = \zeta_i P_A h_i$. Since A_i , τ_i and $\eta_i > 0$ for all $i = 1, \dots, K$, and $\tau_0 > 0$, $\lambda_{\nabla^2 P_i}(2, 1) > 0$ and $\lambda_{\nabla^2 P_i}(3, 1) < 0$. The last inequality follows from $A_i((\tau_0^2 + \tau_i^2)(\eta_i^2 + \tau_i^2))^{\frac{1}{2}} > A_i \eta_i \tau_0$. Therefore, $\nabla^2 P_i(\tau, \eta_i)$ is neither a semidefinite positive nor a semidefinite negative matrix. Consequently, $P_i(\tau, \eta_i)$ is neither a convex nor a concave function of (τ, η_i) . $\square$

From Lemma 1, it follows that the equation (3c) does not represent a convex set. Hence, the optimization problem is nonlinear and non-convex.

IV. OPTIMAL ALGORITHM

Problem (3) is *difficult* to solve because of its non-convexity and nonlinearity. To achieve the optimal solution, we transform the problem into bi-level optimization. We treat the K -user network as K networks with a single user and find an expression for the individual IT times of the users for a fixed EH time. Then, the optimal EH time can be searched with bisection method as the sum of the EH and IT lengths are proven to be convex over EH time.

A. Optimal Solution

Problem (3) is separable into K *easy* and convex subproblems with the objective function and the variable τ_i and constraints regarding only U_i for all $i = 1, \dots, K$, where τ_0 is the only coupling variable between subproblems. Then, the optimal solution to the master problem on τ_0 with the objective function as the sum of the objectives of the subproblems and τ_0 gives the optimal solution to the original problem [11].

All of the subproblems are in the same reduced form of Problem (3) for $K = 1$ and fixed τ_0 . To solve these subproblems, we exploit the nature of a single user network. Note that, for $K = 1$, τ_0 and τ_1 is adjusted just to meet U_1 's requirements. Thus, only the amount of energy required for the UL IT is harvested, i.e., $\eta_1 = 1$.

- For $K = 1$ and $\eta_1 = 1$ (or fixed τ_0), $R([\tau_0, \tau_1, \eta_1])$ defined in (2), turns into a nonlinear, concave function of $[\tau_0, \tau_1]$ (or of $[\tau_1, \eta_1]$) [2], (3b) represents a convex set, and the objective function and (3c) are linear. Thus, both of these reduced problems of (3) are nonlinear and convex.
- Due to the convexity of the reduced problems and their linear objective functions, their optimal solutions will lie on the boundary of their feasible region [12] and satisfy (3b) with equality [8].

Definition 1. τ_0^l and τ_1^l denote the values of τ_0 and τ_1 , respectively, as the solution to the system consisting of (3b) and (3c), both at equality, for $K = 1$ and $\eta_1 = 1$,

$$\tau_0^l = \frac{P_1^{max} D_1}{W \zeta_1 P_A h_1 \log_2 \left(\frac{P_1^{max} \gamma_1}{\zeta_1 P_A h_1} + 1 \right)}, \quad (5)$$

$$\tau_1^l = \frac{D_1}{W \log_2 \left(\frac{P_1^{max} \gamma_1}{P_A \zeta_1 h_1} + 1 \right)}. \quad (6)$$

In the system assumed in Definition 1, U_1 transmits at P_1^{max} in the UL. Hence, τ_0^l represents a critical point for the case where τ_0 is fixed, and η_1 is varying. Any $\tau_0 > \tau_0^l$ will result in an excess amount of energy for U_1 , and η_1 is adjusted to optimize UL transmit power while keeping τ_1 the same, as opposed to the lower τ_0 , where U_1 uses all the harvested energy and τ_1 is increased to meet the data constraint. This is more formally expressed as follows:

Lemma 2. Problem (3) is solved for $\tau_0 = \bar{\tau}_0$ and $K = 1$. For $\bar{\tau}_0 \geq \beta_1$, where $\beta_1 = \tau_0^l$, the optimal solution is

$$\bar{\eta}_1 = \frac{P_1^{max}}{\zeta_1 P_A h_1 \bar{\tau}_0} \bar{\tau}_1, \quad (7)$$

$$\bar{\tau}_1 = \tau_1^l. \quad (8)$$

For $\bar{\tau}_0 < \beta_1$, $\bar{\eta}_1$ is fixed to 1, the optimization problem is reduced to a nonlinear equation,

$$\frac{(2^{D_1/W\bar{\tau}_1} - 1)\bar{\tau}_1}{\gamma_1} = \bar{\tau}_0, \quad (9)$$

where $\gamma_1 = \frac{g_1 \zeta_1 P_A h_1}{\sigma^2}$.

Proof. The problem has two decision variables, but, η_1 is redundant and the trade off between τ_1 and η_1 is irrelevant.

The solution to the system consisting of (3b) and (3c) for fixed τ_0 is (7) and (8). By substituting (8) into (7) and solving it for $\bar{\tau}_0$, we get $\bar{\tau}_0 = \beta_1$ as expected. If $\bar{\tau}_0 < \beta_1$ in (7), then $\eta_1 > 1$, and this corner point is not feasible any more; otherwise, the optimal solution is found.

If the solution above is infeasible, then η_1 is fixed to 1, forcing all harvested energy to be used in the IT. Hence, the optimal solution is found on the intersection point of (3b) with $\eta_1 = 1$. We rewrite (3b) such that η_1 is a function of τ_1 and equate it to 1, and get the expression in (9). Since, the solution to (9) is feasible already, it becomes the optimal solution to the problem. $\square$

Theorem 1. Let $g(\bar{\tau}_0) = \bar{\tau}_0 + \sum_i \bar{\tau}_i(\bar{\tau}_0)$. $g(\bar{\tau}_0)$ is convex over $\bar{\tau}_0$.

Proof. We prove that $\bar{\tau}_1(\bar{\tau}_0)$ is convex. Then, $g(\bar{\tau}_0)$ is also convex as $\bar{\tau}_i(\bar{\tau}_0)$ s are all in the same form, and the convexity is closed under sum.

By Theorem 2.4.1(iii) in [13], $\bar{\tau}_1(\bar{\tau}_0)$ is convex if (1) $\bar{\tau}_1(\bar{\tau}_0)$ is proper, (2) the domain of the function denoted by $\text{dom}\bar{\tau}_1$, i.e. the set of all possible $\bar{\tau}_0$, is convex, and (3) $\bar{\tau}_1(\bar{\tau}_0)$ is subdifferentiable at every $\bar{\tau}_0 \in \text{dom}\bar{\tau}_1$.

- 1) $\bar{\tau}_1(\bar{\tau}_0) : (0, +\infty) \rightarrow (0, +\infty)$, is proper, since $\text{dom}\bar{\tau}_1 = (0, +\infty) \neq \emptyset$.
- 2) $\text{dom}\bar{\tau}_1$ is convex.
- 3) $\partial\bar{\tau}_1(\bar{\tau}_0)$ represents the subdifferential of $\bar{\tau}_1$ at $\bar{\tau}_0$, and it is defined as $\partial\bar{\tau}_1(\bar{\tau}_0) = \{m | m^T(y - \bar{\tau}_0) \leq \bar{\tau}_1(y) - \bar{\tau}_1(\bar{\tau}_0), \forall y \in \text{dom}\bar{\tau}_1\}$ and $\text{dom}\partial\bar{\tau}_1(x) = \{x \in \text{dom}\bar{\tau}_1 | \partial\bar{\tau}_1 \neq \emptyset\}$.

For $\bar{\tau}_0 \neq \beta_1$, $\bar{\tau}_1$ is differentiable. Hence, the subdifferential of the function exists, it is unique and equal to the derivative of the function.

At $\bar{\tau}_0 = \beta_1$, the derivative does not exist. To determine $\partial\bar{\tau}_1(\beta_1)$, we calculate subgradients of $\bar{\tau}_1$ at β_1 , since $\partial\bar{\tau}_1(\beta_1)$ is the set of all subgradients, m 's, defined by the inequality $m(y - \beta_1) \leq \bar{\tau}_1(y) - \bar{\tau}_1(\beta_1)$ for all y .

For $y \geq \beta_1$, $\bar{\tau}_1(y)$ is a constant function and the subgradient inequality is equivalent to $m(y - \beta_1) \leq 0$. $(y - \beta_1) > 0$, then, $m \leq 0$.

For $y < \beta_1$, $\bar{\tau}_1(y)$ is a decreasing function, thus, $\bar{\tau}_1(y) - \bar{\tau}_1(\beta_1) > 0$. Let $\mu_1 < 0$ be a variable that is equal to $\frac{\bar{\tau}_1(y) - \bar{\tau}_1(\beta_1)}{y - \beta_1}$. Then, $m \geq \mu_1$.

Therefore, we have $\partial\bar{\tau}_1(\beta_1) = [\mu_1, 0]$. That means $\bar{\tau}_1$ is subdifferentiable at every $x \in \text{dom}\bar{\tau}_1$, i.e., $\text{dom}\bar{\tau}_1 = \text{dom}\partial\bar{\tau}_1$.

In consequence of (1)-(3), $\bar{\tau}_1(\bar{\tau}_0)$ is convex. Thus, $g(\bar{\tau}_0)$, the sum of convex functions, is convex. $\square$

B. Algorithm

Algorithm 1 Power Constrained Multiple User Time Minimization Algorithm (POWMU)

- 1: Compute: β_i and $\beta_{max} = \max_i \{\beta_i\}$ for all $i = 1, \dots, K$.
- 2: Express: $g(\bar{\tau}_0) = \bar{\tau}_0 + \sum_i \bar{\tau}_i(\bar{\tau}_0)$ for all $i = 1, \dots, K$.
- 3: Search on τ_0 for optimal by bisection method:
- 4: Set $lb = \max_i \{D_i \log(2)/(W\gamma_i)\}$ and $ub = \beta_{max}$.
- 5: **while** $ub - lb > 2\epsilon$ **do**
- 6: $\tau_0' = \frac{lb + ub}{2}$;
- 7: $\frac{dg(\bar{\tau}_0)}{d(\bar{\tau}_0)}|_{\bar{\tau}_0 = \tau_0'}$
- 8: If $\frac{dg(\bar{\tau}_0)}{d(\bar{\tau}_0)} \geq 0$, reset $ub = \tau_0'$
- 9: If $\frac{dg(\bar{\tau}_0)}{d(\bar{\tau}_0)} \leq 0$, reset $lb = \tau_0'$
- 10: **end while**
- 11: Compute and output $g(\tau_0')$

POWMU, first, calculates the critical points of $\bar{\tau}_i(\bar{\tau}_0)$, β_i , for each $i = 1, \dots, K$. Next, the maximum of the critical points, β_{max} , is found. Total transmission time, $g(\bar{\tau}_0)$, is expressed. We perform a bisection search on $g(\bar{\tau}_0)$. The idea is to iteratively converge to the optimal solution inside a search interval by evaluating the slope of $g(\bar{\tau}_0)$ to check where the solution lies. When $\bar{\tau}_0 > \beta_i$, $\bar{\tau}_i(\bar{\tau}_0)$ is constant for all i . Since the total transmit time is the sum of τ_0 with $\bar{\tau}_i(\bar{\tau}_0)$ for all i , β_{max} is set as the upper bound, ub , of the search interval. The lower bound, lb , is set to $\max_i \{D_i \log(2)/(W\gamma_i)\}$ [10]. The current solution, τ_0' , is set to the geometric mean of the lb and ub . If the first derivative of $g(\bar{\tau}_0)$ calculated at τ_0' is nonnegative, meaning that the function increases at that point, then the ub is reset to τ_0' . Otherwise, the lb is reset to τ_0' . This procedure

continues until the difference between ub and lb is less than 2ϵ . Then, search stops and the scheduling length $g(\bar{\tau}_0)$ computed at the last τ_0' is returned as the optimal solution.

Theorem 2. *Given the initial lb and ub , by bisection search method, to obtain an optimal solution in the error bound, ϵ , the number of iterations, k , should be at least as large as $\log_2(ub - lb) - \log_2\epsilon$.*

Proof. Please refer to [14]. $\square$

With the derivative calculations, $\mathcal{O}(K)$, at each iteration and by Theorem 2, the worst case performance of bisection search is $\mathcal{O}(K \log n)$, where n is the length of the search array. In general, $n \gg K$ and search also dominates $\mathcal{O}(K)$ calculations in the rest of the algorithm. Therefore, the overall complexity of POWMU is logarithmic.

V. HEURISTIC ALGORITHM

We propose a heuristic algorithm, MAX-EH, which provides a suboptimal solution to the multiple user problem by exploiting the optimality conditions of a single user network. MAX-EH finds K -many optimal EH times for K users as if there are K networks with a single user. The maximum among them is set as the network's EH time and having EH length known, IT lengths of each user are calculated accordingly.

A. Analytical Analysis

We have already started solving the Problem (3) for $K = 1$ and $\eta_1 = 1$ in Section IV-A and stated in Lemma 2 that, for $\eta_1 = 1$, the problem is reduced to (9). Next, we will solve (9).

Theorem 3. *The solution to the optimization problem with the objective $(\tau_0 + \tau_1)$ and the constraints (3b), (3e), and $\eta_1 = 1$, denoted as τ_0^k and τ_1^k , given by*

$$\tau_1^k = \frac{D_1 \log(2)}{W \alpha_1}, \quad (10)$$

$$\tau_0^k = \frac{D_1 \log 2}{W \alpha_1 \gamma_1} (10^{\alpha_1} - 1). \quad (11)$$

where $\alpha_1 = \mathcal{W}(\frac{\gamma_1 - 1}{e}) + 1$ and $\mathcal{W}(\cdot)$ is the Lambert W-Function.

Proof. Sketch of the proof is as follows. We will first rewrite (3b) such that τ_0 is a function of τ_1 , which is, in fact, the same with (9) and it will be denoted as $V(\tau_1)$. Next, we will prove that $V(\tau_1)$ is strictly decreasing with respect to τ_1 . When we think of the objective function in the slope-intercept form, e.g. $\tau_0 = -\tau_1 + C_1$ and C_1 is the total transmission time that we want to minimize, one of the lines from the family of parallel lines constructed in this way forms a tangent line to $V(\tau_1)$. The tangent line is unique since $V(\tau_1)$ is strictly decreasing; and the tangent point minimizes the objective function. Hence, τ_1 and τ_0 values corresponding to this unique tangent point are calculated.

Fig. 1. Graphical representation of the constraint (3b).

First, we note that τ_1 is strictly positive as to serve the strictly positive D_1 . The first order derivative of $V(\tau_1)$ is

$$\begin{aligned} \frac{dV(\tau_1)}{d\tau_1} &= \frac{(2^{\frac{D_1}{W\tau_1}} (W\tau_1 - D_1 \log 2) - W\tau_1)}{W\gamma_1\tau_1} \\ &< \frac{W\tau_1 - D_1 \log 2 - W\tau_1}{W\gamma_1\tau_1} < 0. \end{aligned} \quad (12)$$

(12) indicates that the function $V(\tau_1)$ is strictly decreasing with respect to τ_1 .

We illustrate the graphical relationship between the curve $V(\tau_1) = \frac{\tau_1}{\gamma_1} (2^{\frac{D_1}{W\tau_1}} - 1)$ and the line $\tau_0 = -\tau_1 + C_1$ in Fig. 1. Depending on the value of C_1 , the line intersects with the curve either at two points, or at one unique point, or does not intersect at all.

Denote C_1^k as the value of C_1 such that the line $\tau_0 = -\tau_1 + C_1$ has only one intersection point with the curve $V(\tau_1)$, and the corresponding τ_1 , is denoted as τ_1^k . Then, at the unique intersection point we have

$$\left. \frac{dV(\tau_1)}{d\tau_1} \right|_{\tau_1=\tau_1^k} = -1, \quad (13)$$

$$V(\tau_1^k) = -\tau_1^k + C_1^k, \quad (14)$$

From (13), τ_1^k is obtained as in (10). By (14), C_1^k is obtained as

$$C_1^k = \frac{D_1 \log 2}{W \alpha_1} \left(1 + \frac{10^{\alpha_1} - 1}{\gamma_1} \right). \quad (15)$$

The area above the curve, $\tau_1 \geq \frac{\tau_1}{\gamma_1} (2^{\frac{D_1}{W\tau_1}} - 1)$, represents the feasible region on Fig. 1, accordingly,

- When $C_1 < C_1^k$, there is no intersection between the line and the curve, hence, feasibility is not satisfied.
- When $C_1 > C_1^k$, there are two intersection points between the line and the curve.

These derivations and observations prove that the objective is minimized at the unique intersection point of the line and the curve, when $C_1 = C_1^k$. $\square$

Corollary 1. *The optimal solution to the Problem (3) for $K = 1$ is (τ_0^k, τ_1^k) if this corner point solution satisfies the constraint (3c), i.e. it is feasible, by Theorem 3. Otherwise, the optimal solution is (τ_0^l, τ_1^l) given in Definition 1, which is always feasible.*

B. MAX-EH Algorithm

Algorithm 2 MAX-EH

- 1: Compute: τ_0^i for all $i = 1, \dots, K$ by Corollary 1.
- 2: Find $\tau_0^{max} = \max_i \{\tau_0^i\}$ for all $i = 1, \dots, K$.
- 3: Compute: τ_1^i for τ_0^{max} , for all $i = 1, \dots, K$, by Lemma 2.
- 4: Compute and output: $\tau_0^{max} + \sum_i \tau_1^i$ for all $i = 1, \dots, K$.

MAX-EH Algorithm is described in detail as follows: Individual optimal EH times of the users are calculated using Theorem 1, and stored in τ_0^i for all $i = 1, \dots, K$. Then, their maximum is found and stored in τ_0^{max} . This maximum value is set as the network's EH length. Using τ_0^{max} , individual IT lengths of the users are calculated. Their sum with τ_0^{max} is computed as total scheduling length of the network and set as the output. MAX-EH is a linear time algorithm with $\mathcal{O}(K)$.

VI. SIMULATIONS AND NUMERICAL RESULTS

The goal of the simulations is to compare the performances of the proposed algorithms, POWMU and MAX-EH, to Transmission Completion Time-minimized (TCT) Algorithm [10] and to evaluate the effect of the maximum power constraint on the network's performance.

TCT finds the optimal schedule length for HTT network where the users are free to transmit at any power. Hence, TCT might set transmit power of the users higher than P_{max} to shorten IT lengths. In our simulations, we force TCT to obey P_{max} limitation, i.e., for any user allocated to transmit at a power level greater than P_{max} , its transmit power is adjusted to P_{max} .

Simulation results are obtained by averaging the performance over 1000 independent random network topologies in MATLAB. The users are uniformly distributed within a circular area of radius 1 m and the AP is located at the center. The network is simulated for an indoor environment operating at a center frequency of $f_c = 915$ MHz and a bandwidth of $W = 1$ MHz.

The channel attenuation is determined considering both large and small scale statistics. The attenuation due to the large scale statistics is modeled as $PL(d) = PL(d_0) + 10\alpha \log(d/d_0) + Z$, where d is the distance between the user and the AP, $PL(d)$ is the path loss at distance d in decibels, $PL(d_0)$ is the path loss at reference distance $d_0 = 1$ m, α is the path loss exponent [15] and Z is a Gaussian random variable with zero mean and standard deviation σ_z . The small-scale fading is modeled by Rayleigh fading with scale parameter Ω set to the mean power level determined by $PL(d)$ [16].

The simulation parameters are as follows: $\sigma^2 = -50$ dBm, $\sigma_z = 2$ dB, $PL(d_0) = 31.7$ dB, $\alpha = 2$, $\zeta = 0.5$, $D = 50$ bits, $P_{AP} = 4$ W, and $P_{max} = 1$ mW.

Fig. 2 depicts the schedule length of a 5-user network for varying P_A . POWMU reduces the total time up to 4.1%, compared to TCT, and MAX-EH converges to POWMU for higher values of P_A . Similar behaviour is observed for networks with larger number of users.

Fig. 3 indicates the schedule length of a 5-user network for varying P_{max} values. POWMU outperforms TCT up to 6.8%,

Fig. 2. Schedule length (s) vs. transmission power (W) of AP for a 5-user network.

Fig. 3. Schedule length (s) vs. the maximum UL transmit power (W) for a 5-user network

especially for smaller P_{max} . As we loosen that constraint, TCT converges to our solution. It is as expected, since we optimize the time allocations considering the transmit power limitation, while, if allowed, TCT tends to assign higher transmit power to the users for faster IT, compromising EH length in favor of a lower total schedule length. Forcing TCT to use P_{max} may result with increased EH length but not improved IT length. MAX-EH performs closely to the optimum, especially for lower P_{max} .

Fig. 4 presents the schedule length for varying number of users, from 10 to 25. POWMU reduces TCT's solution up to 2.9%; whereas MAX-EH performs closely to POWMU.

In Fig. 5, run-times of the algorithms are compared for different number of user, ranging from 1 to 25. Since MAX-EH is a linear time algorithm, it performs significantly better than logarithmic time POWMU and TCT, especially for large-size networks. Likewise, POWMU surpasses TCT as predicted, considering our algorithm performs the optimal τ_0 search on a narrower interval. POWMU and MAX-EH reduce TCT's runtime 71.5% and 84.1% for $K = 25$, respectively.

Fig. 4. Schedule length (s) vs. the number of the users.

Fig. 5. Run time vs the number of users.

VII. CONCLUSION

We analyzed the minimum length scheduling problem for a HTT network with a half-duplex single antenna AP, and multiple, single antenna users. The EH and IT times, and UL transmit powers are allocated to optimize the scheduling length under minimum data and maximum transmit power limitations of the users. The problem is proven to be nonlinear and non-convex. We first transformed this problem into a bi-level optimization problem, where we exploited the convexity of the subproblems. We derived an expression for the IT lengths as a function of EH length, and then, searched for the optimal EH length that minimizes the total length. An algorithm, POWMU, that provides a near optimal solution within an error bound is proposed. Further, we presented a suboptimal algorithm, MAX-EH, which exploits the features of the single user network to solve the multiple user network. Our algorithms are compared with a benchmark algorithm, TCT, which does not regard the UL transmit power limit. Simulation results showed that POWMU and MAX-EH outperform TCT under tight power conditions and their run-times are significantly better, especially for larger networks.

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