Novel distributed feedback lightwave circuit elements

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ABSTRACT

The devices that are variations of inter-and intracoupled meandering optical waveguides are proposed as the lightwave circuit elements that exhibit distributed feedback. A preliminary transfer matrix method analysis is applied in frequency domain, taking the coupling purely directional and with constant coefficient on geometrically symmetric and anti-symmetric devices. The meandering loop mirror is the building block of all meandering waveguide based lightwave circuit elements. The simplest uncoupled meandering distributed feedback structure exhibits Rabi splitting in the transmittance spectrum. The symmetric and antisymmetric coupled meandering distributed feedback geometries can be utilized as band-pass, Fano, or Lorentzian filters or Rabi splitters. Meandering waveguide distributed feedback structures with a variety of spectral responses can be designed for a variety of lightwave circuit element functions and can be implemented with generality due to the analytic approach taken.

Keywords: coupled resonator induced transparency (CRIT) filter, distributed feedback (DFB), Fabry-Perot resonator (FPR), Fano resonator, hitless filter, Lorentzian filter, Rabi splitter, self coupled optical waveguide (SCOW), tunable power divider.

1. INTRODUCTION

The origin of the loop reflectors dates back to 1988 in the optical fiber form.\textsuperscript{1, 2} Then, the double loop mirrors\textsuperscript{3} were shown to be a fiber analogue of Fabry-Pérot resonator.\textsuperscript{4} Furthermore, compound structures such as distributed feedback (DFB) structures were also investigated right afterwards\textsuperscript{5} in fiber optics. The loop reflectors have been widely used for different applications in modern telecommunications for more than a decade.\textsuperscript{6}

Self-coupled optical waveguides (SCOW) has been recently introduced to integrated lightwave circuits\textsuperscript{7} by modifying the well-known add-drop optical filter.\textsuperscript{8} SCOWs utilize the coherent interference of two modes produced in the resonator by feeding the structure with only one coherent field, hence the name self-coupled optical waveguides. Electromagnetically Induced Transparency (EIT) type of spectrum has also been detected in SCOWs,\textsuperscript{9} simply because the aforementioned structure behaves as an analogue of a combination of a Mach-Zehnder interferometer coupled to a ring resonator.\textsuperscript{10}

Motivated by the meandering type waveguides often appearing in microwave engineering,\textsuperscript{11} we showed that both loop mirrors in fiber optics and SCOWs in integrated photonics belong to the same family of resonators, namely meandering waveguides (MWs). We previously introduced these structures, which exhibit rich transmission spectra that can be used in novel photonic lightwave circuit applications.\textsuperscript{12} These generalized MWs, also named Symmetric and Antisymmetric Meandering Distributed Feedback (SMDFB and AMDFB, respectively) structures whose phase responses are reported here for the first time to our knowledge. Additionally, we showed that generalized MW structures are reducible to their simpler counterparts, e.g. SMDFB reduces to MDFB.
2. GEOMETRY OF THE MEANDERING WAVEGUIDE

The structures studied in this work are variations of meandering waveguides directionally coupled at the points where they bend. Figure 1 shows the genealogy of the meandering waveguides.

The meandering loop mirror (MLM), which is shown to have tunable reflectivity is the building block of all MWs. By cascading two MLMs, one can build Meandering Resonator (MR) that is analogous to a Fabry Pérot Resonator. A natural extension of an MR is a Meandering Distributed Feedback (MDFB), which is an MR populated with more than two MLMs.

We devised new meandering geometries in search of a richer characteristic, by placing neighboring MLMs of MR close enough that they also start coupling at their previously uncoupled ends. This new geometry shows a varying characteristic depending on the way the meandering pattern is finalized, Symmetric and antisymmetric meandering resonators, (SMR and AMR respectively). A similar operation executed on MDFB results in two different structures likewise, coined as symmetric and antisymmetric meandering distributed feedback structures (SMDFB and AMDFB).

3. MEANDERING LOOP MIRROR (MLM)

Shown in Figure 2a, Meandering Loop Mirror (MLM) is a waveguide bent to form a single meander, coupling the propagating fields with an intensity coupling constant of C at its base. MLM is effectively composed of 2 bent sections and 2 straight sections with optical phases of \(2\theta_B\) and \(2\theta_S\), respectively. The incident field \(\vec{E}_i\), visualized rightward propagating, couples across the base, creating a field propagating counter-clockwise \(\vec{E}_{ccw}\), eventually interfering with the clockwise propagating component \(\vec{E}_{cw}\) forming the reflected and transmitted fields \(\vec{E}_r\) and \(\vec{E}_t\).

Without loss of information, these fields can be depicted with their phasor representations, which then, via basic algebra, leads to the following expressions (Eqns. 1) for the transmitted and reflected intensities.
\[ I_r = I_i R \quad (1a) \]
\[ I_t = I_i T \quad (1b) \]

When exactly half of the incident power is coupled at the base of the MLM, the interference causes all of the incident intensity to be reflected back, thus by all means, forming a perfect mirror. The spectrum of MLM is independent of the excitation frequency, given that the frequency is guided effectively. MLM does not have a loop that light can propagate closing on itself. In order to have a frequency selective device, one has to use at least two MLMs.

### 4. MEANDERING RESONATOR (MR)

Meandering Resonator is one of the most fundamental structures that exhibit frequency selection out of MWs. MR is composed of two MLMs separated by a spacer waveguide with an optical phase \( \theta_L \) thereby forming a cavity. Following from the fact that an MLM is effectively a mirror, an MR is analogous to a Fabry Pérot resonator.

The fields within MR are just two sets of fields solved for two MLMs coupled together. When solved for coupling constants \( C_1 \) and \( C_3 \), for the first and second points of coupling, the reflectance and transmittance spectra end up depending on the reflectances (Eqns. 1) of the respective MLMs.

\[
I_r = \frac{R_1 + R_3 + 2\sqrt{R_1 R_3} \cos 2(\theta_B + \theta_L + 2\theta_S)}{1 + R_1 R_3 + 2\sqrt{R_1 R_3} \cos 2(\theta_B + \theta_L + 2\theta_S)} \quad (2a)
\]

\[
I_t = \frac{(1 - R_1)(1 - R_3)}{1 + R_1 R_3 + 2\sqrt{R_1 R_3} \cos 2(\theta_B + \theta_L + 2\theta_S)} \quad (2b)
\]

When we define an optical phase \( \delta = 2(2\theta_B + \theta_L + 2\theta_S) \) the resonances associated with the roundtrip optical phase of the MR, at \( \delta = \pi \), can be spotted in Figure 3a. When the coupling constants are not equal, one obtains transmission spectra as in Figure 3b, where the MR is not completely transparent on resonance.

### 5. MEANDERING DISTRIBUTED FEEDBACK (MDFB)

By cascading \( n \) MLMs as in Figure 2c, one can build a device that exhibits distributed feedback (DFB). This geometry can be analyzed with the transfer matrix method (TMM) by taking MLM as a generating block. One can build a scattering matrix that relates fields of a single MLM with respect to fields of its neighbors.

\[
\begin{pmatrix}
E_r^k \\
E_t^k
\end{pmatrix} = S_k
\begin{pmatrix}
E_r^{k-2} \\
e^{i\phi} E_r^{k+2}
\end{pmatrix}
\]

(3)

Here, \( k \) denotes a single MLM, with \( k + 2 \) and \( k - 2 \) its neighboring MLMs, and \( \phi \) being the optical phase accumulated due to the spacer between MLMs. Relating to our single MLM result, one can write down the scattering matrix with respect to our physical parameters.

\[
S_k = e^{2i(\theta_B + \theta_S)} \begin{pmatrix}
2i\sqrt{C_k} \sqrt{1 - C_k} & 1 - 2C_k \\
1 - 2C_k & 2i\sqrt{C_k} \sqrt{1 - C_k}
\end{pmatrix}
\]

(4)

One can obtain transfer matrix \( M_k \) from scattering matrix \( S_k \), which can be multiplied as

\[
M = M_{2n-1} T \cdots M_1 \quad \text{(5)}
\]

with \( T \) being the matrix that propagates the fields between MLMs with an optical phase of \( \theta_L \) in order to obtain a cumulative transfer matrix that relates the boundary fields of the MDFB. Here we also have \( \delta = 2(2\theta_B + \theta_L + 2\theta_S) \) as the optical phase of the MDFB. Figure 4a shows the spectrum for three cascaded MLMs, as an example. As the number of cascaded elements are increased to 6, transmission spectra as shown in Figure 4b are attainable. Here, it can be noticed that the resonance of the MR at \( \pi \) turns into a stop band of the DFB.
Figure 3: Transmission spectra of Meandering Resonators with denoted parameters.

(a) $C_1 = C_3$

(b) $C_1 \in (0, 0.4), C_3 = 0.4$

Figure 4: Transmission spectrum of an (a) MDFB composed by three cascade MLMs with equal $C_n$s. (b) MDFB composed by six cascade MLMs with equal $C_n$s.
6. SYMMETRIC MEANDERING RESONATOR (SMR)

Symmetric Meandering Resonator (SMR) is composed of two MLMs coupled to each other, so that there is no spacer between the two MLMs, Fig. 5a. We applied the coupled field analysis to SMR in order to obtain its transmission spectrum and phase response for different coupling constants. Here we take \( \delta = 2(\theta_B + \theta_S) \) as the roundtrip optical phase. When the odd coupling constants, \( C_1 \) and \( C_3 \) are set to 0, the structure behaves as a power divider, i.e., a hitless filter, which can be tuned by adjusting the coupling constant between the MLMs, \( C_2 \). SMR transmission spectrum and phase response are plotted in Fig. 6a. The phase is computed as \( \pi \) or \( -\pi \) which are essentially equivalent of our purposes, but since these parameters put the system in an unstable equilibrium, the numerical rounding error becomes prevalent, mimicking a bistable behaviour.

As we perturb the odd coupling constants around \( C_1 = C_3 = 0.5 \), we start to observe Fano lineshapes in the transmission spectra, Fig. 6b. As expected, SMR phase starts to demonstrate sudden changes of \( \sim \pi \) at Fano resonances. As we further move away from the \( C_{1,3} = 0.5 \), Fano resonances push each other away, Fig. 6c. Such a frequency behaviour points out an analogy between the SMR and a resonator enhanced Mach-Zehnder interferometer (REMZI). By increasing the coupling from \( C = 0.6 \) to \( C = 0.9 \), Fig. 6d we observe the evolution of the Fano resonances into a band-stop filter.

7. ANTISYMMETRIC MEANDERING RESONATOR (AMR)

The shortest possible MW resonator one can make is Antisymmetric Meandering Resonator (AMR), which consists of two coupling points. One can extend the output port of the MLM while introducing a feedback via coupling back into the waveguide, creating a resonance interference pathway as seen in Figure 5b. AMR visually resembles a racetrack resonator.

When the field equations derived for the AMR geometry are solved, Equation 6 are obtained for AMR’s transmission and reflection spectra. Here we take \( \delta = 2(\theta_B + \theta_S) \) as the roundtrip optical phase.

\[
I_r = \frac{4(1 - C_1)(1 - C_2)(C_1 + C_2 + 2\sqrt{C_1C_2}\cos(2(\theta_B + \theta_S)))I_i}{(1 + C_1C_2 + 2\sqrt{C_1C_2}\cos(2(\theta_B + \theta_S)))^2} \\
I_t = \frac{(1 + C_1(-2 + C_2) - 2C_2 - 2\sqrt{C_1C_2}\cos(2(\theta_B + \theta_S)))^2}{(1 + C_1C_2 + 2\sqrt{C_1C_2}\cos(2(\theta_B + \theta_S)))^2}I_i
\]

These equations result in a spectrum that have various characteristics, e.g. Lorentzian lineshape and Coupled Resonator Induced Transparency (CRIT) behaviour in different combinations of coupling constants, which are plotted in Figure 7. For example, AMR behaves as a notch filter when the round trip phase is around \( 2\pi \) and \( C = 0.1 \). Then the spectrum evolves to a CRIT filter around \( \pi \) at higher coupling constants.

![Figure 5: (a) Symmetric Meandering Resonator with two MLM’s having coupling constants as \( C_1 \) and \( C_3 \), coupled to each other with \( C_2 \). (b) Geometry of the antisymmetric meandering resonator, AMR.](http://proceedings.spiedigitallibrary.org)
Figure 6: SMR transmittance and phase spectrum (a) $C_1 = C_3 = 0.5$ for $C_2 = 0.2, 0.4, 0.6$ and $0.8$ (b) $C_1 = C_3 = 0.49$ for $C_2 = 0.2, 0.4, 0.6$ and $0.8$ (c) $C_1 = C_2 = C_3 = 0.6, 0.8$ and $0.9$ (d) $C_1 = C_3 = 0.4$ and $C_2 = 0.3, 0.6$ and $0.9$. 
8. SYMMETRIC MDFB (SMDFB) AND ANTISYMMETRIC MDFB (AMDFB)

The meandering distributed feedback (MDFB) structure is the generalization of the meandering resonator (MR) with $n+1$ coupled MLMs. The number of couplings amongst the MLMs turns out to be $2n+1$, when a total of $n+1$ MLMs is used. Unlike in the MDFB structure with uncoupled MLMs, the basic unit in the current structure has 6 ports. SMDFB and AMDFB consist of input and output boundary MLMs (BMLMs) and a finite chain of channel MLMs (CMLMs). Excluding the two BMLMs, all of the CMLMs are similar to each other with 6 ports, i.e., 3 ports to the left and 3 ports to the right, Figs. 8a and 8b.

We obtained the analytical expression of the transmission intensity under certain conditions with applying TMM (Transfer Matrix Method) to the SMDFB and AMDFB structures with 6 ports. For an individual MLM
with 6 ports, the equations read

\[
\begin{pmatrix}
E_k^k \\
E_k^{k-1} \\
E_k^{k+1} \\
E_{ccw}^{k+1} \\
E_{cw}^{k-1} \\
E_{c}^k
\end{pmatrix}_{\text{out}} = S_k
\begin{pmatrix}
E_k^k \\
E_k^{k-1} \\
E_k^{k+1} \\
E_{ccw}^{k+1} \\
E_{cw}^{k-1} \\
E_{c}^k
\end{pmatrix}_{\text{in}}
\]

where these scattering matrix is:

\[
S_k = \frac{2\sqrt{1-C_k} \sqrt{1-C_{k+1}}}{\sqrt{1-C_k} \sqrt{1-C_{k+1}} e^{i\theta_k}} \begin{pmatrix}
2i \sqrt{1-C_{k+1}} e^{i\theta_k} & \sqrt{1-C_k} e^{i\theta_k} & i \sqrt{C_k} e^{i\theta_k} & i \sqrt{C_k} e^{i\theta_k} & \sqrt{1-C_k} e^{i\theta_k} & 2i \sqrt{1-C_{k+1}} e^{i\theta_k} \\
0 & \sqrt{1-C_k} e^{i\theta_k} & 0 & 0 & \sqrt{1-C_k} e^{i\theta_k} & 0 \\
i \sqrt{C_k} e^{i\theta_k} & \sqrt{C_{k+1}} e^{i\theta_k} & 0 & 0 & \sqrt{C_k} e^{i\theta_k} & i \sqrt{C_k} e^{i\theta_k} \\
i \sqrt{C_k} e^{i\theta_k} & i \sqrt{C_k} e^{i\theta_k} & 0 & 0 & i \sqrt{C_k} e^{i\theta_k} & i \sqrt{C_k} e^{i\theta_k} \\
i \sqrt{C_k} e^{i\theta_k} & \sqrt{C_{k+1}} e^{i\theta_k} & i \sqrt{C_k} e^{i\theta_k} & i \sqrt{C_k} e^{i\theta_k} & \sqrt{1-C_k} e^{i\theta_k} & \sqrt{1-C_{k+1}} e^{i\theta_k} \\
\sqrt{1-C_k} e^{i\theta_k} & i \sqrt{C_k} e^{i\theta_k} & i \sqrt{C_k} e^{i\theta_k} & \sqrt{1-C_k} e^{i\theta_k} & \sqrt{1-C_{k+1}} e^{i\theta_k} & \sqrt{1-C_k} e^{i\theta_k}
\end{pmatrix}
\]

Since there is no simple recipe for the transformation from a scattering matrix to a transfer matrix of dimension 6, we will solve the scattering matrix equations for the transfer matrix variables. The transfer matrix equations are:

\[
\begin{pmatrix}
E_{r,out}^{k+1} \\
E_{r,cw,out}^{k+1} \\
E_{r,in}^{k+1} \\
E_{r,cw,in}^{k+1} \\
E_{c}^{k+1} \\
E_{c}^{k+1}
\end{pmatrix} = M_k
\begin{pmatrix}
E_{r,in}^{k-1} \\
E_{cw,in}^{k-1} \\
E_{r,in}^{k-1} \\
E_{cw,in}^{k-1} \\
E_{c}^{k-1} \\
E_{c}^{k-1}
\end{pmatrix}
\]

where the transfer matrix is:

\[
M_k = \begin{pmatrix}
0 & e^{i\theta_k} \sqrt{C_{k+1}} & 0 & e^{i\theta_k} \sqrt{C_{k+1}} & 0 & 0 \\
0 & 0 & e^{i\theta_k} \sqrt{C_{k+1}} & 0 & 0 & 0 \\
e^{i\theta_k} \sqrt{C_k} & 0 & 0 & 0 & 0 & 0 \\
e^{i\theta_k} \sqrt{C_k} & 0 & 0 & 0 & 0 & 0 \\
e^{-i\theta_k} \sqrt{1-C_{k+1}} & 0 & 0 & 0 & 0 & 0 \\
e^{-i\theta_k} \sqrt{1-C_{k+1}} & 0 & 0 & 0 & 0 & 0 \\
0 & e^{i\theta_k} \sqrt{C_{k+1}} & 0 & e^{i\theta_k} \sqrt{C_{k+1}} & 0 & 0 \\
0 & 0 & e^{i\theta_k} \sqrt{C_{k+1}} & 0 & 0 & 0 \\
e^{-i\theta_k} \sqrt{1-C_{k+1}} & 0 & 0 & 0 & 0 & 0 \\
e^{-i\theta_k} \sqrt{1-C_{k+1}} & 0 & 0 & 0 & 0 & 0 \\
é^{-i\theta_k} \sqrt{1-C_{k+1}} & 0 & 0 & 0 & 0 & 0 \\
é^{-i\theta_k} \sqrt{1-C_{k+1}} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

For a SMDFB with \( n \) coupling constants the total transfer matrix is
\[ M = M_{2n-1} T M_{2n-3} T \ldots M_3. \tag{11} \]

The transition matrix between the \((k - 2)\)th MLM and the \(k\)th MLM is

\[
\begin{pmatrix}
E_{cw, in}^{k-1} \\
E_{t, in}^{k-1} \\
E_{r, in}^k \\
E_{cw, out}^k \\
E_{t, out}^{k-1} \\
E_{r, out}^{k-1}
\end{pmatrix} = T
\begin{pmatrix}
E_{r, out}^{k-1} \\
E_{cw, out}^{k-2} \\
E_{t, out}^{k-1} \\
E_{r, out}^{k-1} \\
E_{cw, in}^{k-2} \\
E_{r, in}^k
\end{pmatrix}, \tag{12}
\]

where the transition matrix is

\[
T = \begin{pmatrix}
i & 0 & 0 & 0 & 0 & 0 \\
0 & i & 0 & 0 & 0 & 0 \\
0 & 0 & e^{i\theta_B} & 0 & 0 & 0 \\
0 & 0 & 0 & -i & 0 & 0 \\
0 & 0 & 0 & 0 & -i & 0 \\
0 & 0 & 0 & 0 & 0 & e^{-i\theta_B}
\end{pmatrix}. \tag{13}
\]

We analyzed the limiting cases of a SMDFB with 5 coupling points. Here we take \(\delta = 2(\theta_B + \theta_S)\) as shortest optical path phase difference. Figure 9a shows that SMDFB can be reduced to MDFB with uncoupled MLMs, in the limiting case where the even indicated coupling constants are taken to be zero. Since the optical roundtrip path phase difference of MDFB with uncoupled MLMs is twice of that of SMDFB when \(L = 0, 2\delta = 4(\theta_B + \theta_S)\). Figure 9b points out the case with \(C_i \rightarrow 0\) and \(C_i \rightarrow 1\) where \(C_i\) runs from \(i = 1 : 5\), which shows almost total transparency as expected. Figure 9c shows the case with \(C_i \rightarrow 1, C_j \rightarrow 0\) where \(i : Z_{even}\) and \(j : Z_{odd}\) and vice versa, which again reduces the meandering resonator to a delay line.

The same analysis might be applied to the AMDFB by utilizing the transfer matrix under different right side boundary equations. The transmission spectra and their corresponding phase responses for the limiting cases, namely \(C_i \rightarrow 0\) and \(C_i \rightarrow 1\) where \(C_i\) runs from \(i = 1 : 4\) and \(C_i \rightarrow 1, C_j \rightarrow 0\) where \(i : Z_{even}\) and \(j : Z_{odd}\) and vice versa, can be seen in Fig. 10a. We can also reduce AMDFB structure to a Meandering Resonator with an additional phase, when we use \(C_{odd} = 0.1, 0.2, 0.4\) and \(C_{even} \rightarrow 0\), Fig. 10b.

9. CONCLUSIONS

The spectra and phases of meandering waveguides (MW) are analyzed in the frequency domain. The meandering loop mirror (MLM) is the building block of all meandering waveguide structures. Two MLM’s make a meandering resonator (MR). Multiple MLM’s behave as meandering distributed feedback (MDFB) structures, which exhibit Rabi splitting in the transmittance spectrum. The analysis of the frequency response of the symmetric and antisymmetric resonators, symmetric and antisymmetric MDFB’s structures with coupled MLM’s results in the following observations: The symmetric MR (SMR) could be utilized as a tunable power divider or a tunable hitless filter by adjusting the value of \(C_2\), when \(C_1\) and \(C_3 = 0.5\). SMR could be also used as a Fano resonator, when different coupling constants are used with also having sudden changes in its phase responses. SMR has different spectral regions, which show Fano resonances, notch and CRIT filters behaviours from region to region. The antisymmetric MR (AMR) could be utilized as a CRIT filter starting from a specific state of coupling constants, around \(C_1 = C_2 = 0.7\). Similarly AMR also demonstrates both CRIT and notch filters behaviour at different regions. The symmetric MDFB (SMDFB) and antisymmetric MDFB (AMDFB) could be utilized as delay lines at certain limit coupling states. Under specific conditions, SMDFB and AMDFB can be reduced to a MDFB and an MR, respectively.
Figure 9: Transmittance spectrum of an SMDFB structure (a) with 5 coupling points when $C_2 = C_4 = 0.001$ and $C_1 = C_3 = C_5 = 0.1, 0.2, 0.4$. (b) with 5 coupling points for $C = 0.01$ and 0.99 for all coupling points. (c) with 5 coupling points when $C_{even} = 0.01, C_{odd} = 0.99$, and $C_{odd} = 0.01, C_{even} = 0.99$. 
Figure 10: Transmittance spectrum of a AMDFB structure (a) with 4 coupling points when $C_i = 0.01$ and $C_i = 0.99$ for $i = 1:4$ and when $C_i = 0.01, C_j = 0.99$ where $i : Z_{even}$ and $j : Z_{odd}$ and vice versa. (b) with 4 coupling points when $C_i = 0.1, 0.2, 0.4$ and $C_j \rightarrow 0$ where $i : Z_{even}$ and $j : Z_{odd}$ and vice versa.

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