Micro-mirror-array based off-axis flat lens for near-eye displays

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Abstract: We developed an off-axis diffractive lens using a micro-mirror array on a flat substrate. MMA creates an on-axis converging beam from a 45 degrees off-axis diverging illumination beam and functions similar to a large and bulky elliptical mirror. The array consists of individual micro-mirrors with normal directions that vary across the component. The size, normal direction and the center height of each micro-mirror are optimized to achieve a phase matching condition so that the smallest focal spot size is achieved at the design wavelength. Design can also be optimized for full color applications using a synthetic design wavelength. A sample MMA of size 3 mm by 5 mm is fabricated using grayscale lithography. The designed MMA is used to illuminate a computer-generated hologram in a near-eye display system. Experimental results verify the premises of the designed component.

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1. Introduction

In the design of augmented reality (AR) near-eye displays, it is desired to have a wide field-of-view (FOV) and small form factor [1]. Conventional AR head-worn displays with wide FOV utilize bulky optical lenses and mirrors as relay components. The poor form factor of such displays impedes their acceptance in consumer applications. Alternative approaches for reducing the size and bulk of the optics have exploited different solutions. Active pupil tracking [2] is one of the promising solutions, but the optical mechanisms used in tracking the pupil are complicated. Unconventional relay optics including holographic optical elements [3], waveguides [4] and diffractive optics [5] are other approaches to reduce the form-factor in near-eye displays.

Holographic optical components (HOEs) using volume gratings constitute a good candidate for replacing bulky optical elements with thin and flat components and thus make the optics of near-eye displays more compact and lightweight [6]. However, HOEs are fabricated by recording interference patterns from two coherent light beams on high-resolution photosensitive materials. This process requires carefully aligned and stable optical setups for each wavelength and optical design. Furthermore, for some components such as high NA lenses, physically obtaining the wavefronts that are free of aberrations is difficult and may require the physical fabrication of the associated conventional component first. In some cases, the wavelength of recording laser does not match the desired wavelength, further complicating the fabrication process [7]. The shrinkage of the final HOEs should also be considered and compensated during the fabrication process [8].

Micro-mirror arrays (MMA) discussed in this paper constitute an alternative flat lens technology with certain advantages over HOEs. To begin with, the MMA does not require a physical interference setup to be prepared as in the HOE case but can be numerically designed to replace any conventional optical relay element for any desired wavelength. While a HOE requires each new sample to be separately recorded in the interference setup, an initial MMA sample can be fabricated with grayscale lithography for once and then can be used as a master shim via which many replicas of the MMA can be easily fabricated. Furthermore, on the contrary to HOEs where light efficiency falls rapidly with deviations from design
wavelengths and input light directions, the MMA, due to its reflective nature, preserves its light efficiency with minor changes in the output focal spot position and shape.

Active MMA designs that have previously been studied mostly provide poor quality due to the large micro-mirror sizes [9]. Several passive MMAs are designed based on ray optics and used in display technologies [10,11]. Obtaining high diffraction efficiency is a challenge in design and fabrication especially for diffractive optical elements (DOEs) with large deflection angles. Such DOE’s include off-axis and high numerical aperture lenses. To obtain high efficiency, the phase function of the DOE’s is either approximated by a multilevel structure or realized directly as continuous surface relief with analog profile [12].

MMA is essentially a type of surface relief DOE with a discrete pixelated structure. In this paper, an off-axis diffractive lens using an MMA on a flat substrate is designed based on wave-optics simulations. The designed MMA is used to illuminate a computer-generated hologram in a near-eye display system. The normal directions and center heights of each micro-mirror are optimized to minimize the size of the focused spot. The center height of each micro-mirror has a nanometer scale difference with respect to the adjacent micro-mirrors to satisfy a phase matching condition, where the fields interfere constructively at the focal plane. The experimental results show that the designed and fabricated compact MMA can replace a bulky elliptical optical relay.

The MMA design process is discussed in Section 2. A small section of the designed MMA is selected and fabricated using gray-scale lithography and the results are detailed in Section 3.

2. Design process of MMA

MMA discussed in this paper consist of flat micro-mirrors and is designed for the optical configurations shown in Fig. 1. Schematic drawings of the perspective view of the imaging and near-eye setup are illustrated in Figs. 1(a) and 1(c), respectively. The distances of the imaging and near-eye setups are illustrated in Figs. 1(b) and 1(d), respectively. In these illustrations, the optical axis is parallel to the z axis and a point light source is placed at (3, 0, 3) cm from the origin. The light emanating from the light source is reflected back after hitting the MMA and gets focused at (0, 0, 12) cm. The performance of the MMA is tested using a binary computer generated hologram (CGH). As illustrated in Figs. 1(c) and 1(d), the reflective binary mask is placed at (0, 0, 6) cm. The rays reflect back from the binary mask and get focused on the user’s pupil plane after reflecting from a beam-splitter.

![Fig. 1. Optical setups utilizing the MMA. (a) Perspective schematic drawing and (b) top view of the imaging setup illustrating the object and image distances. (c) Perspective schematic](image-url)
To obtain the best performance, the MMA must be designed (i.e., the normal direction and position of each micro-mirror should be adjusted) taking into account wave optics and interference effects. Failure to do so results in systems with quite poor performance. We first go through an analysis where the interference among reflections from different micro-mirrors is ignored or has a random nature. We show that in that case the achieved focal spot size is unacceptably large. Then, we show that once mirrors are designed so that reflections form constructive interference at the focal plane, the achieved focal spot size is drastically minimized.

When the reflections from the micro-mirrors of the MMA interfere with a random nature (with arbitrary phases), the focal spot size on the observation plane cannot be smaller than the size of the beam generated by a single micro-mirror. Further, the focus is achieved only when all mirrors are tilted toward the focus spot. Assuming this is the case, we analyze the spot size generated by a single micro-mirror. A drawing of a micro-mirror of the MMA is provided in Fig. 2. The dashed red line shows a chief ray, which emanates from the source and reflects back from the center of the micro-mirror and continues its path toward the observation plane. The dotted line is the image of the illumination part due to the flat micro-mirror. The green arrows in this figure indicate the distances used in equations. The aperture projection of each MMA with size $\Delta$ is referred as $\Delta_N$, where $\Delta_N = \Delta \cos \theta$.

![Fig. 2. A drawing of the corner micro-mirror of the MMA. The dashed red line shows chief ray that reflects through the center of the micro-mirror.](image)

The geometrical boundary of the spot ($D_G$) on the observation plane can be found approximately using the following relation:

$$D_G = \Delta_N \frac{d_i + d_s}{d_s},$$

(1)

where $d_i$ is the image distance between the micro-mirror center and the focus point, $d_s$ is the distance between the point light source and the center of the micro-mirror, as shown in Fig. 2. Assuming the micro-mirror has a square cross-section, the enlargement in the size of the reflected spot due to diffraction ($D_D$) can be estimated using:

$$D_D = 2.44 \frac{d_i}{\Delta_N},$$

(2)
where $\lambda$ is the wavelength. Therefore, the non-optimized size of the focused spot on the pupil plane is $D_G + D_d$. A minimum spot size of 1.68 mm can be achieved using flat micro-mirrors with a pitch size of 220 $\mu$m, which is not acceptable for near-eye applications. Note that even if the micro-mirrors were curved so that geometric contribution to the spot size was zero, due to diffraction and random interference of the fields emanating from micro-mirrors, an acceptable spot size will not be achieved. In addition, curved profiles with correct optical power and quality are much more difficult to fabricate.

We focused mainly on micro-fabricated small micro-mirrors where flat profiles produce a diffraction-limited performance. In our design, the micro-mirrors are placed such that the projection of the micro-mirror centers on the plane $x$-$y$ is equally spaced by an amount of $\Delta$ and have a square shape. Note that other regular tessellations (e.g., hexagons) could be used instead but rectangular form is easier to design and easier to implement in the layout. Further, the diffraction orders due to the pixelated nature of the MMA with square shaped micro-mirrors are placed in a rectangular grid, which is more suitable for the near-eye application as it provides a rectangular eye-box, which can be filtered out more easily.

To specify the tilt of each micro-mirror, we will refer to the angle of the normal vector with $x$, $y$ and $z$ axes. The range of the tilt of each individual micro-mirror in the final MMA will be between 20 degrees to 40 degrees. Illustrations in Figs. 3(a) and 3(b) show the three-dimensional (3D) MMA top view and zoomed views of the corner micro-mirrors.

One convenient way to fabricate the MMA is using the gray-scale lithography techniques. Etch depth ($h_{\text{max}}$), which is limited to less than 30 microns in our process, sets a maximum limit for the micro-mirror size. The peak-to-peak height of the corner micro-mirror is plotted with a solid blue line in Fig. 4(a). As evident from Fig. 4(a), the size of each individual micro-mirror in the array must not exceed 60 microns. The other limiting factor in the design of an MMA is the separation of the 0th and 1st diffraction orders ($D_0$), which can be calculated using:

$$D_0 = \frac{\lambda d}{\Delta}. \quad (3)$$

If the MMA is used in a near-eye display, the separation between diffraction orders at the pupil plane must be greater than the eye pupil diameter size, which is taken as 2 mm for our design (indicated with a solid red line in Fig. 4(b)). The separation of diffraction orders with respect to the micro-mirror pitch size is plotted in Fig. 4(b) with a solid blue color. Based on the simulations in Fig. 4, the size of each micro-mirror in the array must be less than 40 microns. Single curved mirror solution is not a flat component and as such the main
The motivation of this paper is to use an array of micro-mirrors. We selected a micro-mirror size of 40 microns in order to balance several factors. Smaller mirrors are desired to have sufficient space between diffraction orders at the desired distance and to keep the maximum required etch depth due to tilt of the mirror within the microfabrication limits. Larger mirrors up to 40 microns are better to minimize the edge effects and other tolerances due to microfabrication.

**Fig. 4.** (a) In our fabrication process $h_{\text{max}}$ is restricted to be less than 30 microns (indicated with a solid red line), which sets a maximum limit for the micro-mirror size. (b) The distance between diffraction orders with respect to the micro-mirror pitch size. If the MMA is used in a near-eye display, the separation between diffraction orders at the pupil plane must be greater than the eye pupil diameter size, which is taken as 2 mm for our design and shown with a solid red line.

Total field generated by the MMA (within scalar wave optics theory of light) can be expressed as

$$U(r) = \sum_{m} U_{m}(\overline{R}_{m} \cdot r + t_{m}), \quad (4)$$

where $r \in \mathbb{R}^{3 \times 1}$ is the coordinates of the observation point in global coordinate system, $\overline{R}_{m} \in \mathbb{R}^{3 \times 3}$ and $t_{m} \in \mathbb{R}^{3 \times 1}$ are the rotation matrix and the translation vector to transform to the local coordinate system of the $m^{th}$ micro-mirror. In the equation above, $U_{m}$ is the function describing the light generated by $m^{th}$ micro-mirror in the local coordinates of the $m^{th}$ micro-mirror, where it assumed that x and y axes lie parallel to the mirror surface, z axis is aligned in the normal direction of the mirror, and origin is placed the center of micro-mirror. Using Rayleigh-Sommerfeld diffraction integral, the expression for $U_{m}$ can be written as

$$U_{m}(r) = \frac{z}{j\lambda} \int \frac{e^{j(x'y - x'z)}}{|r' - r'_m|} \frac{e^{j(r \cdot r')}}{|r - r'|} dr', \quad (5)$$

where $r'_m$ denotes the position of the virtual image of the point light source formed by the $m^{th}$ micro-mirror in the local coordinates of the $m^{th}$ micro-mirror. In the design of MMA, we want to form a focal spot at $r_f$, i.e., we want to ensure that $|U(r)|^2$ has a global maximum and a strong peak at $r = r_f$. We optimize the center heights and normal directions (tilt angles)
of each micro-mirror to achieve this goal. The changes in the center height and normal directions of each micro-mirror affects $U(r)$ through the $R_m$, $t_m$ and $r_m$ terms.

Rather than pursuing an optimization based on the complicated formulas above, we follow an intuitive approach, which is verified by our simulations and experimental results. Our approach has two assumptions. First, we assume that $|U(r)|^2$ will have a peak at $r = r_f$, if at $r = r_f$, all individual contributions $U_m(R_mr_f + t_m)$ have the same phase value, that leads to a full constructive interference. When this condition is met at $r = r_f$, as $r$ gets away from $r_f$, the full constructive interference condition gets violated due to changing phase values, resulting in a destructive interference and a rapid fall in the intensity that ensures the peak at $r = r_f$. Second, we assume that phase of $U_m(R_mr_f + t_m)$ is equal to $\frac{2\pi}{\lambda}(|r - r_m| + |r_m - r_f|)$ and the full constructive interference condition will be met if $|r - r_m| + |r_m - r_f| = k\lambda$, ($k \in Z$), where $r_m$ is the global coordinate of the $m^{th}$ micro-mirror’s center and $r_f$ is the global coordinate of the point light source. Strictly speaking, the 2nd assumption holds if diffraction due to the mirror aperture is ignored (which is not possible in our case). Nonetheless, assuming that tilt angles of mirrors will be adjusted to steer the light from the source towards the focal point, the approximation is valid to a good extent.

To find the field distribution in the focus point, we used wave optics simulation of the architecture provided in Fig. 1 using MATLAB. In this model, the size of the MMA is assumed to be 5 mm x 3 mm. In our simulations, we assign an aperture for each micro-mirror to model the pitch size and shadows, a virtual prism (grating) that handles the steering of the light towards the focus point and a phase offset to ensure that we have all constructive interface at $r_f$. Using this model, we can find the field on the entire aperture of MMA and we free-space propagate the field on MMA towards the focal spot.

Figure 5 demonstrates the simulated far-field point spread function (PSF) of the MMA at the pupil plane. The intensity ($I$) of the signal $U$ is $I = |U|^2$. The normalized intensity ($I_n$) is calculated as $I_n = I / \max(I)$ and used in plots. Figure 5(a) shows the spot size without phase-matching condition for the monochrome case ($\lambda = 632$ nm). The PSF is distributed over x axis with a diameter of almost 1.5 mm in Fig. 5(a) because the fields from individual micro-mirrors interfere randomly at the pupil plane. Random interference problem can be mitigated by optimizing the normal directions and center heights of each micro-mirror. The center height of each micro-mirror has a nanometer scale difference with respect to the adjacent micro-mirrors to satisfy the phase matching condition, where the fields interfere constructively at the focal plane. Optimizing the center heights ensures that the optical path from the light source to the focus point becomes an integer multiple of the design wavelength and the phase matching condition is satisfied.

Figures 5(b) and 5(c) demonstrate the spot size where the phase-matching condition is satisfied for a single wavelength. The plots in Figs. 5(b) and 5(c) show the same normalized intensity plot with different horizontal axis (x axis) scales. The focused spot size is reduced to approximately 30 microns after satisfying the phase matching condition, as evident in Fig. 5(c). The full-width half-maximum of the simulated focused spot in Fig. 5(c) is 10.4 microns. The maximum center height difference between two adjacent micro-mirrors after applying the phase matching condition is 96 nm for the monochrome MMA design.

For full-color applications, a synthetic design wavelength is defined as the least common multiple of red, green and blue wavelengths. This synthetic wavelength can be used for optimizing the center heights of micro-mirrors. Figure 5(d) shows the spot size for the full-color case without phase matching condition. The red, green and blue wavelengths are chosen as 632, 520 and 450 nm. Figures 5(e) and 5(f) demonstrate the spot size where the phase-
matching condition is satisfied for the red, green and blue wavelengths. The plots in Figs. 5(e) and 5(f) show the same normalized intensity plot with different horizontal axis (x axis) scales. As evident in Figs. 5(f) and 5(e), the separation distance between the 0th and the 1st diffraction orders reduced in the full-color case since the blue color has a smaller wavelength compared to the red wavelength. This issue can be solved by reducing the size of the micro-mirror pitch from 40 microns to 20 microns. The maximum center height difference between two adjacent micro-mirrors after applying the phase matching condition is 238 nm for the full-color MMA design.

![Fig. 5.](image)

Fig. 5. The far-field point spread function (PSF) of the MMA at the pupil plane. (a) shows the spot size without phase-matching condition for the monochrome case with $\lambda = 632$ nm. (b) and (c) demonstrate the spot size where the phase-matching condition is satisfied for a single wavelength. The plots (b) and (c) show the same normalized intensity plot with different horizontal axis (x axis) scales. (d) shows the spot size for the full-color case without phase matching condition. The red, green and blue wavelengths are chosen as 632, 520 and 450 nm. (e) and (f) demonstrate the spot size where the phase-matching condition is satisfied for the red, green and blue wavelengths. The plots (e) and (f) show the same normalized intensity plot with different horizontal axis (x axis) scales.
The schematic drawing of center positions before enforcing the phase-matching condition are illustrated in Fig. 6(a). Figure 6(b) shows the schematic drawing of modified center positions after enforcing the phase-matching condition. The variation of the center positions along the MMA aperture for the monochrome case is illustrated in Fig. 6(c). As evident in Fig. 6(c), the maximum center height difference between two adjacent micro-mirrors after applying the phase matching condition is 96 nm for the monochrome MMA design. The cross section of the plot provided in Fig. 6(c) with a vertical micro-mirror index of 55 is illustrated in Fig. 6(d).

To achieve good optical performance, surface roughness should be less than $\lambda / 20$ across any 2mm x 2mm area (corresponding to pupil area), and the total roughness across the whole part should be less than $\lambda / 2$ to ensure good flatness across the component.

MMA creates an on-axis converging beam from a 45 degrees off-axis diverging illumination beam as shown in Fig. 7(a). The size of the diffraction limited focused spot on the pupil plane is simulated by propagating a converging wave passing through a 5 mm single slit (as shown in Fig. 7(b)). In Fig. 7(c), the logarithmic plot of the focused spot PSF created with the phase-matched MMA is indicated with a solid red line and the size of the diffraction limited focused spot on the pupil plane is plotted with a solid blue line. As evident in Fig. 7, the MMA acts as a diffraction limited component after satisfying the phase matching condition.
3. Experimental results

To perform the experiments, we fabricated a sample part with a size of 3 mm by 5 mm using gray-scale lithography on a photo-resist polymer placed on a 1 mm thick N-BK7 glass substrate. The high cost of fabricating a custom sample part was the main limiting factor on the size of the MMA. The MMA area is surrounded by a 250 µm border that has no exposure. Around this is another border area with a trench. The trench is nominally 5 µm deep and 100 µm wide. The side length of the square micro-mirrors is 40 microns. Center of each micro-mirror has a different height in the order of a few nanometers. The final part is nickel coated photoresist master with a thickness of 30 microns. Figure 8 shows the 50x measurement of the fabricated MMA (parts were fabricated and measured by Infinite Graphics Co, USA).

Fig. 7. (a) MMA creates an on-axis converging beam from a 45 degrees off-axis diverging illumination beam. (b) The size of the diffraction limited focused spot on the pupil plane is simulated by propagating a converging wave passing through a 5 mm single slit. (c) The logarithmic plot of the focused spot PSF created with the phase-matched MMA is indicated with a solid red line and the size of the diffraction limited focused spot on the pupil plane is plotted with a solid blue line. As evident in the plot, the MMA acts as a diffraction limited component after satisfying the phase matching condition.

Fig. 8. The 50x measurement of the fabricated MMA. (a) The height difference between two corners of a single micro-mirror is 29.2 µm. After removing tilt, the height variation across a single micro-mirror is measured to be 90 nm. (b) A better representation of the fabricated part, where the height reduces as we move from blue parts toward the red parts. (Fabricated and measured by Infinite Graphics Co., USA).
To perform this measurement, the tilt had been removed and the height variation is measured using MicroSpy scanning chromatic confocal surface measuring instrument. As evident in Fig. 8(a), the height difference between two corners of the micro-mirror at the corner is 29.2 \( \mu \text{m} \). After removing tilt, the height variation across a single micro-mirror due to the step artifacts of gray-scale lithography process is measured to be 90 nm, which means that the RMS surface roughness of the fabricated micro-mirrors meets the optical quality requirements. Figure 8(b) provides a better representation of the fabricated part, where the height reduces as we move from blue parts toward the red parts.

Fig. 9 illustrates the focus spot created by the fabricated MMA using the setup provided in Figs. 1(a) and 1(b). Due to the periodic structure of the fabricated micro-mirrors, instead of a single focus point, multiple replicas (diffraction orders) of the focus spot is observed at the focus plane, as expected. Based on our wave optic simulations, if there was no phase matching condition, the diffracted waves reflected from each micro-mirror would not interfere constructively and the focused spot would be as large as the space between the diffraction orders (in millimeter range). As evident from the focused spot pictures, the phase matching condition is satisfied as desired and the spot size is much smaller than the diffraction order separation. The full-width at half-maximum is measured as 40 microns, as shown in Fig. 9. The value of the experimental PSF is larger than the simulated PSF due to the step artifacts of the lithography process. Diffraction orders appear due to the periodic nature of the design. Fourier filtering or using non-periodic designs are techniques to eliminate diffraction orders at the expense of reduced contrast.

The MMA is used in the architecture shown in Fig. 10(a). A binary CGH mask is illuminated using the fabricated MMA. The static binary amplitude CGH used in this experiment is computed using the procedure discussed in [13] and the encoding is performed using the error diffusion algorithm instead of iterative Fourier transform algorithm. A 1.5 mm x 2.5 mm area of the mask is illuminated and the reconstructed image is captured using a CCD camera. The captured image is illustrated in Fig. 10(b). The field of view is limited by the MMA to 1.43° by 2.38° and the distance between the eye and the reconstructed image is 500 mm. The size of the captured image is 12.5 mm x 20.8 mm at a distance of 500 mm from the eye. This experiment proves that such an MMA can be used in a near-eye display system with coherent light illumination.
Fig. 10. (a) The near-eye display architecture used in the experiment with 45 degrees off-axis incidence from the source. The distances in the setup are reported in Fig. 1(d). (b) Image of the reconstructed hologram captured with a camera at the location of the eye. The size of the captured image is 12.5 mm x 20.8 mm at a distance of 500 mm from the eye.

4. Conclusion

A micro-mirror array (MMA) as an alternative thin and flat lens technology is introduced to reduce the size and bulk of the relay optics for an augmented reality (AR) head-worn display application. The MMA discussed in this paper is an off-axis diffractive lens with certain advantages over holographic optical elements. The MMA can be numerically designed for any desired wavelength. A computer-generated design is used together with the gray-scale lithography techniques to fabricate MMA. An initial MMA sample can be fabricated with grayscale lithography for once and then can be used as a master shim via which many replicas of the MMA can be easily fabricated. This array consists of individual square micro-mirrors with different normal directions and center heights. The micro-mirrors in the array need to have different center heights in the order of a few nanometers to satisfy the phase matching condition at the design wavelength. For full-color applications, a synthetic design wavelength is defined as the least common multiple of red, green and blue wavelengths. This synthetic wavelength can be used for optimizing the center heights of micro-mirrors. In our implementation, MMA creates an on-axis converging beam from a 45 degree-off-axis diverging illumination wave. The designed MMA functions similar to a large and bulky elliptical mirror. Due to its reflective nature, the MMA preserves its light efficiency with minor changes in the output focal spot and shape. Micro-mirror size is optimized using diffraction simulations, considering fabrication constraints, and to satisfy the phase-matching condition. A sample part of size 3 mm by 5 mm is fabricated using gray-scale lithography on a photo-resist polymer placed on a 1 mm thick N-BK7 glass substrate. The side length of each individual square micro-mirror is 40 microns. The fabricated MMA is used in a near-eye display architecture. The field of view is limited by the MMA to 1.43° by 2.38°. This experiment proves that such an MMA can be used in a near-eye AR display system with highly off-axis coherent light illumination.

Funding

European Research Council Seventh Framework Programme (FP7/2007-2013) ERC advanced grant agreement no 340200, Wear3D.
Acknowledgments

We are grateful to Mr. Mark Mulhern (Infinite Graphics and Imaging (IGI) company) for his help with the fabrication and characterization.

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