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Çinlar Model at Fully Developed Channel Flow for $Re_\tau = 2000$

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Abstract. Large eddy simulation (LES), which is the numerical solution method of Navier-Stokes equation, is performed in a fully developed channel flow. In LES, Çinlar and homogeneous dynamic Smagorinsky models are used as subgrid-scale models and the results are compared with direct numerical solution (DNS). A channel flow at Reynolds number $Re_\tau = 2000$ is computed using $192 \times 64 \times 48$ mesh resolution. The results of Çinlar model are in good agreement with especially those of homogeneous dynamic Smagorinsky, even if the size of the grid is much coarser than the DNS resolution.

INTRODUCTION

In fluid dynamics, the turbulent motion is modelled by the Navier-Stokes equations (NSE). Direct numerical solution (DNS) of the NSE necessitate to take an account all the scales, small and large. So, the computational cost of this procedure is considerably high. A numerical solution technique is large eddy simulation (LES), in which case the effect of the small scales on the larger scales is modelled using resolved velocity components. For separating large scales from small ones, a filter is applied to NSE and a nonlinear term called the subgrid scale stress (SGS), which needs to be modelled, come into sight in LES equations. It can be decomposed into three parts, the Reynolds stress, the cross stress and the Leonard stress. The Reynolds stress represents interaction of the subgrid scales, the cross stress represents interactions of the resolved and the subgrid scales, and the Leonard stress involves only the resolved velocity, which is computed explicitly.

Several SGS models have been proposed, among them Lilly [13, 14], Deardorff [7], Leonard [12] and Smagorinsky [16] are the pioneers. Usually, a subgrid scale viscosity ν_{sgs} is introduced for modeling the effect of small scales motion on the large ones. In this study, we consider dynamic Smagorinsky model, which is based on eddy viscosity idea, and Çinlar SGS model, which is based on physical interpretation of the subgrid scales interaction. The latter is derived by using Çinlar random velocity field, which is based on vortex structures [5]. Çinlar velocity field is a proper stochastic model for small to medium scale turbulent flow, see e.g. [1, 2, 3, 4]. Moreover it has been shown that the velocity field can capture the subgrid scale energy spectrum in [11]. Inspired by vortex stretching model of Misra and Pullin [15], we have formed the Reynolds stress using the covariance tensor of Çinlar velocity field to develop our SGS model in [10]. For numerical computation, OpenFOAM, which is an open source simulation program for computational fluid dynamics, is used considering turbulent channel flow. Çinlar SGS model is compared with dynamic Smagorinsky for $Re_\tau = 2000$.

The paper is organized as follows. First, LES and dynamic Smagorinsky as a benchmark SGS model are reviewed. Then, Çinlar SGS model is described. Finally, numerical results for turbulent channel flow are given.

LARGE EDDY SIMULATION

In LES, flow variables are decomposed into the resolved (filtered) and the unresolved subgrid scale terms by the help of a filter function. The velocity field can be decomposed as $u(x) = \bar{u}(x) + u'(x)$ where $\bar{u}(x) = \int_{-\infty}^{\infty} u(\xi)G(x - \xi)d^3\xi$ is

the filtered velocity field in space and G is the filter function that determines the size and structure of the small scales. After the filtering operation which is applied to incompressible Navier-Stokes equations, the filtered Navier-Stokes equations are obtained as

$$\begin{aligned}\frac{\partial \bar{u}_i}{\partial x_i} &= 0 \\ \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} &= -\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial}{\partial x_j} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \frac{\partial \tau_{ij}}{\partial x_j}\end{aligned}\quad (1)$$

where $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ is subgrid scale tensor and it must be modelled to represent the effect of small scales.

Subgrid stress tensor is decomposed as $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j = L_{ij} + C_{ij} + R_{ij}$ where $L_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ is the Leonard tensor, which represents interactions among large scales [12], $R_{ij} = \overline{u'_i u'_j}$ is the Reynolds stress term, which represents interactions among the small scales, and $C_{ij} = \overline{u_i u'_j} + \overline{u'_j u_i}$ is the cross term, which represents the cross-scale interactions between the resolved and unresolved scales.

The SGS turbulence models usually use the eddy viscosity idea of Boussinesq's theory as a basis, which indicates that the subgrid stress tensor is proportional to the resolved strain rate : $\tau_{ij} = -2\nu_t \bar{S}_{ij}$ where $\bar{S}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$ is the resolved strain rate tensor, and ν_t is the eddy viscosity.

One of the commonly used SGS models is Smagorinsky model based on eddy viscosity idea [16]. The eddy viscosity is modelled by $\nu_t = (C_s \bar{\Delta})^2 |\bar{S}|$ where C_s is Smagorinsky constant, $|\bar{S}|^2 = 2\bar{S}_{ij}\bar{S}_{ij}$ is the magnitude of the strain rate tensor, $\bar{\Delta}$ is the grid size. Another eddy viscosity model, which is called dynamic Smagorinsky, based on computation of constant C_s with a dynamic procedure and a second filter, which is known as test filter and denoted by $\hat{\cdot}$, is applied as follows $C_s = \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle}$ where $\langle \cdot \rangle$ is the volume average and $M_{ij} = 2\bar{\Delta}^2 |\bar{S}| \hat{S}_{ij} - 2\hat{\Delta}^2 |\hat{S}| \hat{S}_{ij}$ [8].

ÇINLAR REYNOLDS STRESS MODEL

Çinlar SGS model is based on Çinlar velocity field for subgrid scale and make use of the viscous effects with the resolved strain tensor. The generalized form of Çinlar velocity field is constructed as a random sum of vortices given by

$$u'(x, t) = \int_{-\infty}^t \int_Q N(ds, dz, da, db) e^{-c_q(t-s)} a v\left(\frac{x-z}{b}\right) \quad (2)$$

where $x \in \mathbb{R}^2$, $t \in \mathbb{R}$, $q = (z, a, b)$ represents the type of an eddy and includes its center z in space, its amplitude a as well as its radius b and N is Poisson random measure on the Borel sets of $\mathbb{R} \times Q$ with mean measure $\mu(dt, dq) \equiv \mu(dt, dz, da, db) = \lambda dt dz \alpha(da) \beta(db)$ where λ is the arrival rate per unit time-unit space, and α and β are probability distributions for the amplitudes and radii of eddies, respectively. The notation u' is used to indicate that we aim to model the subgrid scales with (2). The decay parameter is explicitly given by $c_q(x) = c \left| \frac{x-z}{b} \right|^{2\gamma}$ where $c > 0$ and $\gamma > 0$ [4]. $v = (v_1, v_2)$ is the deterministic velocity field, called the basic eddy, on $\mathbb{R}^2$ and $Q = \mathbb{R}^2 \times \mathbb{R} \times (0, \infty)$ be the set of types of eddies. The basic vortex v is taken as a rotation on $\mathbb{R}^2$, with center at 0.

For modeling Reynolds stress, which represents the interaction of the small scales, the covariance tensor of the velocity field is computed as

$$R^{ij}(x, t) = \frac{\lambda}{c} \int_{\mathbb{R}^2} dz \int_{\mathbb{R}} \alpha(da) a^2 \int_{\mathbb{R}_+} \beta(db) \frac{b^2 \exp(-c|z|^{2\gamma}|t|)}{|z|^{2\gamma} + |z + \frac{x}{b}|^{2\gamma}} v_i(z) v_j\left(z + \frac{x}{b}\right)$$

The covariance at space and time lag (0,0) is used as an approximation for the Reynolds stress R_{ij} . It is assumed that the vortex structures have random orientations in three dimensions, which is the same point of view with Misra and Pullin [15]. For orienting Çinlar vortices in a three dimensional flow, we can approximate $R_{ij} \approx (\delta_{ij} - e_i e_j) R^{11}(0, 0)$. Assuming that the vortices are aligned with the resolved vorticity vector $\bar{\omega} = \nabla \times \bar{u}$ we model $e_i \approx \bar{\omega}_i / |\bar{\omega}|$ as in [15, pg.2445]. As a result, we set

$$R_{ij} \approx \left(\delta_{ij} - \frac{\bar{\omega}_i \bar{\omega}_j}{|\bar{\omega}|^2} \right) \frac{\lambda}{c} \mathbb{E}(a^2) \mathbb{E}(b^2) \frac{3\pi}{8}$$

as the Reynolds stress, and model its parameters with the resolved velocity field as given in [10]. As a result, Reynolds stress is modelled as

$$R_{ij} \equiv \frac{3\pi}{32} \frac{C_3^2 C_4}{C_1} \bar{u}^2 \bar{\Delta}^2 e^{-2C_1|\bar{S}|} \cdot \left(\delta_{ij} - \frac{\bar{\omega}_i \bar{\omega}_j}{|\bar{\omega}|^2} \right) \left[\theta(\theta + 1) - \frac{C_3^{-\theta}}{e^{1/C_3} \Gamma_{1/C_3}(\theta)} \left(\frac{1}{C_3} + \theta + 1 \right) \right]$$

where the radius of the largest eddy in dissipation range B is taken to be equal to half of the grid size as $\bar{\Delta}/2$, and $\theta := C_2 \bar{\Delta}/(|\bar{S}| \bar{u})$ will be used. For the cross stress C_{ij} , we use Clark's cross stress model [6], and Leonard stress L_{ij} does not need to be modelled as it depends only on the resolved velocity field. These altogether form the subgrid scale tensor τ_{ij} .

NUMERICAL RESULTS

In this section, the channel flow results of the LES simulation with two different SGS models, namely, Çinlar and dynamic Smagorinsky, are compared with the DNS performed by Jimenez et al. [9] for friction Reynolds numbers of 2000. LES is performed by using the OpenFOAM CFD Toolbox [17]. The computational mesh is $192 \times 64 \times 48$, for $Re_\tau = 2000$ whereas the mesh for DNS was $6114 \times 633 \times 4608$. The channel size is $8\pi\delta \times 2\delta \times 3\pi\delta$. The periodic boundary condition in the streamwise and spanwise directions, and no-slip boundary condition on the wall have been applied in channel flow and both subgrid scale models are implemented with a van Driest damping function. The friction velocity u_τ is used for normalizing the numerical results.

FIGURE 1. Channel flow comparison data for $Re_\tau = 2000$ along the non-dimensional distance y^+ , (a) mean velocity profile U^+ , (b) ms velocity fluctuation streamwise direction u^+ ; (c) ms velocity fluctuation wall-normal direction v^+ , (d) ms velocity fluctuation spanwise direction w^+

Our model is tested by comparing with DNS data for Reynolds number 2000 and dynamic Smagorinsky SGS model in a fully developed channel flow. The numerical results show us that LES with Çinlar model is in as good

agreement with the DNS data as dynamic Smagorinsky even though a very coarse mesh is used in LES simulation. In Fig.1 (a), the mean velocity is plotted, rms velocity fluctuations are given in Fig.1 (b-d). The discrepancy between DNS and LES results originate from mesh grid size which is much coarser in LES than DNS. Dynamic Smagorinsky and Çinlar SGS models show almost the same profile especially in Fig.1 a)and (d), and produce similar results in this case.

CONCLUSION

In this study, a recently proposed SGS model based on Çinlar velocity field has been studied for incompressible flows at a high Reynolds number. The Reynolds stress tensor, which represents the interaction of the small scales, has been modeled using this vortex based velocity field, while the cross-term is taken from the Clark model. The model has been tested by comparing with DNS data and dynamic Smagorinsky SGS model in a fully developed channel flow. The numerical results show that LES with Çinlar model is in as good agreement with the DNS data as dynamic Smagorinsky model, even with a coarse mesh.

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