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Citation: Journal of Mathematical Physics 44, 943 (2003); doi: 10.1063/1.1540714

View online: http://dx.doi.org/10.1063/1.1540714

View Table of Contents: http://aip.scitation.org/toc/jmp/44/2

Published by the American Institute of Physics
Erratum: Pseudo-Hermiticity for a class of nondiagonalizable Hamiltonians

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[DOI: 10.1063/1.1540714]

Recently, the authors of Ref. 1 used the framework provided in Ref. 2 to re-examine the consequences of pseudo-Hermiticity for the class of block-diagonalizable Hamiltonians introduced in Ref. 2. In doing so, they discovered that Theorem 2 of Ref. 2 did not hold, as they could find a counter-example. This theorem must be replaced with the following.

Theorem 2: Let $H$ be as in Theorem 1 of Ref. 2. Then $H$ is pseudo-Hermitian if and only if it is Hermitian with respect to an inner product $\langle \langle \cdot, \cdot \rangle \rangle$ that supports a positive-semidefinite basis including the eigenvectors of $H$. In particular, for every eigenvector $\psi$ of $H$, $\langle \langle \psi | \psi \rangle \rangle \geq 0$; if the corresponding eigenvalue is real and nondefective (algebraic and geometric multiplicities are equal), $\langle \langle \psi | \psi \rangle \rangle > 0$; otherwise $\langle \langle \psi | \psi \rangle \rangle = 0$.

Proof: As shown in Ref. 2, pseudo-Hermiticity of $H$ implies that $H$ is Hermitian with respect to the inner product $\langle \langle \cdot, \cdot \rangle \rangle_\eta$ with $\eta$ given by Eq. (15) of Ref. 2 and $\sigma_{n,a}^1 = 1$. It is not difficult to check that indeed the basis vectors $|\psi_{n,a,i}\rangle$, constructed in Ref. 2, have the property that $\langle \langle \psi_{n,a} | \psi_{n,a} \rangle \rangle = 0$, and that $\langle \langle \psi_{n,a} | \psi_{n,a} \rangle \rangle > 0$ only for the cases that $p_{n,a} = 1$ and $E_n \in \mathbb{R}$, i.e., $|\psi_{n,a,i} = 1\rangle$ is an eigenvector of $H$ with a real eigenvalue. Furthermore, by construction, this basis includes all the eigenvectors of $H$. The proof of the converse is the same as the one given in Ref. 2.

It is important to note that having a positive-semidefinite basis does not imply that the inner product $\langle \langle \cdot, \cdot \rangle \rangle_\eta$ is positive-semidefinite (unless the Hamiltonian is diagonalizable and has a real spectrum in which case both the basis and the inner product $\langle \langle \cdot, \cdot \rangle \rangle_\eta$ are positive definite). If the Hamiltonian has defective or complex-conjugate pair(s) of eigenvalues, there will always be at least two null vectors with negative linear combinations. Unlike positive vectors, linear combinations of nonnegative vectors need not be nonnegative.

3 A vector $\phi$ is respectively said to be positive, null (zero), negative, if $\langle \langle \phi | \phi \rangle \rangle > 0$, $\langle \langle \phi | \phi \rangle \rangle = 0$, $\langle \langle \phi | \phi \rangle \rangle < 0$. It is said to be nonnegative if $\langle \langle \phi | \phi \rangle \rangle \geq 0$. A basis is called positive-semidefinite if it consists of nonnegative vectors. See, for example, J. Bognar, Indefinite Inner Product Spaces (Springer, Berlin, 1974).

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