Wave optics analysis of corner-cube retro-reflectors in near-to-eye displays based on scanning laser projectors

Seyedmahdi Kazempourradi, Erdem Ulusoy, Sven Holmstrom, and Hakan Urey

Department of Electrical and Electronics Engineering, Koç University, Istanbul, Turkey;

ABSTRACT

In near to eye displays based on scanning laser projectors, retro-reflectors seem as convenient image relay components since they can ideally be placed at any location on the scanned beam path. In case of practical retro reflectors though, such as corner cube retro-reflectors (CCRs), the relayed image suffers from loss in quality and resolution due to the positional shift in the retro-reflected rays and the diffraction effects. We perform a wave optics simulation to analyze the image relay performance of a CCR. Our model assumes that the scanned spot of the projector is imaged by the CCR into an array of spots, which superpose and interfere to yield the effective scan spot seen by an eye looking at the CCR. The results indicate that the CCR results in a significant broadened spot size. Experimental results verify the simulation model in terms of achievable resolution and image quality.

Keywords: Wave Optics and Diffraction Simulations, Retro-reflector, Scanning Laser Projectors

1. INTRODUCTION

Retro-reflectors are extensively used in various optical applications, such as traffic signs¹ and laser ranging.² Scanning laser projectors³ are combined with retro-reflectors in various display applications, such as head-up displays, head-mounted displays, and near-to-eye displays to get unique advantages. In near-to-eye displays, retro-reflectors provide flexibility in optical system design since they can ideally be placed at any location on the scanned beam path. Many aspects of retro-reflectors have been studied in the literature. A simple model of retro-reflector is provided by Eckhardt.⁴ Poizat *et al.* explored the use of retro-reflective sheets in optical systems.⁵

A retro-reflector reflects an incoming ray parallel to itself, but with a positional shift rather than along the same line. Moreover, the finite apertures of retro-reflectors mirrors introduces diffraction. As a result, especially in a near-to-eye display, the obtained image suffers some loss in quality and resolution. That is why a satisfactory wave optics analysis, especially within the context of a scanning laser projector based near-to-eye display application is required.

A quantitative description of retro-reflector effect and its artefacts on the image quality is provided by Hua *et al.* using a ray-tracing model.⁶ Although the diffraction effect has a significant impact on image quality, there is no full wave optics analysis available in the literature. Most models, estimate the spot enlargement due to the diffraction by the reported $\frac{4.08\lambda d}{a}$ formula,⁶ where λ is the wavelength, d is the distance from image plane to retro-reflector screen, and a is the aperture size. Such analysis is incomplete since each diffraction aperture is treated individually, and the interference effects between beam segments falling on different apertures are not taken into account.

Here we perform a wave optics simulation to understand the effects of using a corner-cube retro-reflector (CCR) as the relay component in a scanning laser projector based near-to-eye display. The outline of the paper is as follows: We describe the near-to-eye system that utilizes a retro-reflector screen. Afterwards, we study the imaging properties of CCRs. We provide the simulation and experimental results and we conclude the work.

Further author information:

Seyedmahdi Kazempourradi: E-mail: mkazempour14@ku.edu.tr

Figure 1. Schematic of the studied near-to-eye display. The laser scanning projector (LSD) scans a beam of laser light which would project an image on the image plane if the CCR were not present. The scanned laser beam is reflected by the CCR towards the LSD. The beam splitter directs the reflected beam towards the eye. The system effectively images the aperture of the LSD on the eye pupil and creates a scanned retinal image which ideally appears at the distance of the image plane.

2. SYSTEM DESCRIPTION

We will consider a near-to-eye display in this paper and Fig. 1 illustrates the schematic of this setup. An image is projected by the laser scanning projector and passes through the beam splitter (BS). The retro-reflector screen reflects the incoming light toward BS. The BS reflects the light toward the exit pupil (EXP) of the system. The user can see projected images by placing the eye in the EXP of the system.

The retro-reflector screen does not necessarily coincide with the image plane. A system designer can benefit from this feature to come up with compact designs, which is extremely important in near-to-eye and head-worn displays.

3. IMAGING PROPERTIES OF RETRO-REFLECTORS

In this section, we study the imaging properties of retro-reflectors within the configuration depicted in Figure 1 and we focus on corner-cube retro-reflectors (CCRs). We start with analyzing a single CCR cell based on a ray optics model and then we discuss the wave optics features of a single CCR cell. Afterwards, we study the imaging properties of a complete CCR structure.

3.1 Simple Model of a CCR cell

A CCR cell can be made of three mutually perpendicular plane mirror surfaces. The three axes of the coordinate system can be defined as u_1 , u_2 , and u_3 . We choose u_1 , u_2 , and u_3 such that each plane of the CCR cell lies perpendicular to one of the axes.⁴

We can span the incoming ray $\bar{\mathbf{r}}_i$ into

$$\bar{\mathbf{r}}_i = r_{i1}\hat{\mathbf{u}}_1 + r_{i2}\hat{\mathbf{u}}_2 + r_{i3}\hat{\mathbf{u}}_3. \quad (1)$$

This ray reflects back toward plane 2 after impinging the plane 1 (perpendicular to axis u_1) and we can write:

$$\bar{\mathbf{r}}_1 = -r_{i1}\hat{\mathbf{u}}_1 + r_{i2}\hat{\mathbf{u}}_2 + r_{i3}\hat{\mathbf{u}}_3. \quad (2)$$

Figure 2. Incident ray and output ray of a single CCR cell. The red ray ($\bar{\mathbf{r}}_i$) enters the CCR cell and after three internal reflections exits as the green ray ($\bar{\mathbf{r}}_o$).

Similarly, the ray reflected from the second and third plane will be

$$\bar{\mathbf{r}}_2 = -r_{i1}\hat{\mathbf{u}}_1 - r_{i2}\hat{\mathbf{u}}_2 + r_{i3}\hat{\mathbf{u}}_3, \quad (3)$$

$$\bar{\mathbf{r}}_3 = -r_{i1}\hat{\mathbf{u}}_1 - r_{i2}\hat{\mathbf{u}}_2 - r_{i3}\hat{\mathbf{u}}_3. \quad (4)$$

After three reflection, we will have the output ray, i.e., $\bar{\mathbf{r}}_3 = \bar{\mathbf{r}}_o = -\bar{\mathbf{r}}_i$. In other words, the output rays are the ones which experience three reflections.

As we can see in Fig. 2, the reflected ray is parallel to the incident ray. Since all ray bundles cannot experience three reflections, some of the incoming rays are reflected back to an undesired direction. The desired portion of the reflected ray bundles can be considered the same as would be caused by a finite-sized plane mirror.

To characterize the spot size a CCR cell, we refer to Fig. 3. In Fig. 3, the green point targeted by the projector. The red point I_s is the image of green point created by planes of the CCR cell. To characterize the spot size, we assume that a ray bundle comes out of I_s and passes the CCR cell (red solid lines). The incoming rays are diffracted at the CCR cell because the CCR cell has a finite size. Dashed red lines in Fig. 3 define the geometrical spot size and the dashed blue lines define the boundaries of the diffracted spot. We will use this model to simulate the retro-reflective screens.

3.2 Properties of a CCR Screen

The main difference of imaging properties of a CCR screen compared to a single CCR cell is that there will be multiple image points (cf. Fig. 4). One important issue in analyzing CCR screen is that the ray bundles coming from different image points actually interfere with each other so that simple formulas based on a single CCR cell for estimating the diffracted spot size are not accurate. In our model, we start from the object point and we find the image replicas. Afterwards, we propagate the portion of the ray bundles emitted from these image points that passes through the effective aperture of the corresponding CCR cell. The individual wave-fields are superposed, and the effective scan spot (as seen by an eye looking at the CCR) is obtained as the resulting interference pattern.

In this paper, we provide a wave optics simulation of CCR screens. By means of our model, we can study the diffraction effects of a CCR screen in more details. To model a CCR screen, we start from the object points and we find the image replicas of these image points. Afterwards, we only propagate the portion of the ray bundle omitting from these image points that passes through the effective aperture of the corresponding CCR cell.

Figure 3. Spreading of the retro-reflected beam due to geometrical optics and diffraction. Green rays are emitted by the projector to generate a pixel at the image plane. These rays are blocked by the CCR cell and reflected backwards, as shown by the red solid lines, to form a real image of the scan spot. Dashed red lines determine the effective geometrical spot and the dashed blue lines determine the effective diffracted spot.

Figure 4. Geometrical optics demonstration of a CCR screen in a 2D plane. The red points ($I_{s1}, \dots, I_{s4}$) are the image point replicas of the object point.

Figure 5. Spot size simulation results of a CCR screen with $150\ \mu\text{m}$ pitch. Plots (a) and (b) show the same results and only x -axis scale is different in these two plots. The blue curve shows the spot shape in the presence of the CCR screen. The red curve illustrates the spot shape in the absence of CCR screen. Both plots are normalized to have equal energy.

4. SIMULATION AND EXPERIMENTAL RESULTS

Using our model, we studied the effect of CCRs on spot size and spot shape. We also performed experiments of the near-to-eye setup to examine the capability of CCR screen in near-to-eye applications.

4.1 Simulation of the Image Spot Size

As we discussed in the previous sections, the image spot size enlarges due to geometrical nature of CCR and the diffraction effect. The geometrical optics behavior of CCR screens have been explored extensively. Here, we focus on the wave optics simulations.

In these simulations, we assume that the object consists of only one point. By applying our model, we will explore the spot size and spot shape.

Figure 5 presents the spot size simulation results of a CCR screen with $150\ \mu\text{m}$ pitch. The retro-reflector screen is 1 mm in width. We choose the size because of the laser scanning projector mirror size. The distance between object point and CCR sheet is 47 cm and the distance between exit aperture of scanner and the CCR sheet is 3 cm. Figures 5(a) and (b) show the same results and only x -axis scale is different in these two plots. The blue curve shows the spot shape in the presence of the CCR screen. The red curve illustrates the spot shape in the absence of CCR screen. As evident in Fig. 5, using retro-reflector increases the spot size and it distorts the spot shape, which degrades the image quality.

Figure 6 demonstrates the spot size simulation results of a CCR screen with $250\ \mu\text{m}$ pitch. The setup is similar to the previous one. Figures 6(a) and (b) report the same results and only x -axis scale is different in these two figures.

An important observation from Fig. 5 and Fig. 6 is that the spots at the image plane do not have a simple pattern and this fact proves that simple formulas do not work for analysing CCR behavior in near-to-eye displays. Also it is obvious that the diffraction effects the image resolution dramatically. One reason for having an enlarged spot with a distorted shape is that the fields emitted from image points arrive image plane with different phases. When the phases are not matched with each other, the interference of them leads to a spot similar to the ones shown in blue curves in Fig. 5 and Fig. 6.

Figure 6. Spot size simulation results of a CCR screen with $250\ \mu\text{m}$ pitch. Plots (a) and (b) show the same results and only x -axis scale is different in these two plots. The blue curve shows the spot shape in the presence of the CCR screen. The red curve illustrates the spot shape in the absence of CCR screen. Both plots are normalized to have equal energy.

4.2 Experiments of the Near-to-eye Setup

In order to verify the simulations, we perform experiments with the setup shown in Fig. 1. We use a lens and a camera as an eye model.

Figure 7 shows the experimental results using a CCR screen with $150\ \mu\text{m}$ pitch. Images in Fig. 7(a), (b), and (c) are obtained when the CCR screen coincides the image plane. Then, we moved the CCR screen toward eye and placed it in a distance of 10 cm from eye (and laser scanning projector consequently). As it was expected, we encounter a blurred image. These images are illustrated in Fig. 7(d), (e), and (f).

To check the effect of pixel pitch on the image quality, we performed the same experiment with a CCR screen of pitch $250\ \mu\text{m}$. Images in Fig. 8(a), (b), and (c) are captured when the CCR screen is placed on the image plane. Afterwards, we moved the CCR screen toward eye and placed it in a distance of 10 cm from eye (and laser scanning projector consequently). The obtained images are illustrated in Fig. 7(d), (e), and (f).

The quality of the images obtained from CCR screen with larger pixel pitch is better in near-to-eye display. The reason is that the diffraction effect is less dominant when we have larger pitch. It is also observed that the image quality degrades when we bring the CCR screen near to eye. Due to diffraction, the energy in the images spreads and we obtain a lower contrast level.

5. CONCLUSION

In this paper we present a model that enables wave optics analysis of corner-cube retro-reflectors (CCRs). We assume that the CCR creates a multitude of real images of the virtual focus spot in front of the CCR. Diffraction effects introduced by each CCR cell are accounted for by confining the numerical aperture of the corresponding real image point. Our analysis indicates that the effective pixel spot size depends on the phase difference that is introduced by each real image point arriving on the focus plane. In other words, it is not possible to obtain nice images with a retro-reflector even when CCR pitch decreases down to a few microns. The experimental and simulation results are in agreement in terms of achievable resolution and image quality.

Figure 7. Experimental results with a CCR screen of pitch $150\ \mu\text{m}$. Images (a), (b), and (c) are obtained when the CCR screen coincides the image plane. Images (d), (e), and (f) are captured when the CCR screen is placed in a distance of 10 cm from eye.

Figure 8. Experimental results with a CCR screen of pitch $250\ \mu\text{m}$. Images (a), (b), and (c) are obtained when the CCR screen coincides the image plane. Images (d), (e), and (f) are captured when the CCR screen is placed in a distance of 10 cm from eye.

ACKNOWLEDGMENTS

Financial support from ERC Wear3D Project No: EC. 00062 is gratefully acknowledged.

REFERENCES

- [1] W. Su and D. Li, “Retroreflectivity and night visibility of road traffic signs,” *J. Highway and Transp. Res. and Dev.* **26**, 114–119 (2009).
- [2] D. Currie, S. D. Agnello and G. D. Monache, “A lunar laser ranging retroreflector array for the 21st century,” *Acta Astronautica* **68** 667–680 (2011).
- [3] S. Holmstrom, U. Baran, H. Urey, “MEMS laser scanners: a review,” *J. Microelectromechanical Sys.* **23** 259–275 (2014).
- [4] H. D. Eckhardt, “Simple model of corner reflector phenomena,” *Appl. Opt.* **10**, 559–1566 (1971).
- [5] D. Poizat and J. P. Rolland, “Use of retro-reflective sheets in optical system design,” *Tech. Rep. TR98-006* (1998).
- [6] H. Hua, A. Girardot, C. Gao, and J. P. Rolland, “Engineering of head-mounted projective displays,” *Appl. Opt.* **39**, 3814–3824 (2000).