Modelling and Analysis of a Business Model to Offer Energy Saving Technologies as a Service

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In this study, we present a stochastic model to analyze a business plan that is based on offering energy saving technologies as a service. In this arrangement, the total differential cost of replacing an existing technology with a more efficient one is financed from the future energy savings that are shared between a service provider that installs the more efficient technology and its customer. The model we present captures improvements in energy efficiencies and costs of technologies with time, variation in energy consumption, uncertainty in energy prices and useful life of a technology, and revenue from carbon offsets. By using an analytical model, we analyze the feasibility of this business model by using expected cost and also value-at-risk criteria. We show that when the service provider selects the contract parameters in a right way, the business plan brings financial benefits. The customer also benefits financially from reduction in energy usage and replacement costs, and also from additional revenue obtained through selling carbon offsets. Furthermore, since this business plan is based on increasing energy efficiency, the proposed approach decreases energy consumption, and therefore carbon dioxide emissions. As a result, by using an analytical model, we show that offering energy saving technologies as a service is a win-win-win situation for the service provider, its customer, and for the environment.

Key words: Energy Efficiency, Business Model, Technology Replacement, Stochastic Model

1. Introduction

The energy used by manufacturing sector to convert raw materials into finished products is quite significant. In United States, it accounts for over 25% of the total energy usage and 28% of the carbon dioxide (CO_2) emissions. Moreover, around 40% of the energy used onsite at the manufacturers directly or indirectly is lost (Brueske et al. 2012). As a result, increasing energy efficiency of the technologies used in manufacturing is an important way for the manufacturers to increase their

competitiveness, for energy sustainability, and also for the environment.

On the other hand, replacement costs of the existing technologies with more efficient ones may inhibit a burden for manufacturers to undertake these energy efficiency improvements (Muthulingam et al. 2013). Therefore implementing energy efficiency improvement projects requires finding a feasible way to finance these projects (Rezessy and Bertoldi 2010). Since decreasing energy consumption by replacing an existing technology with a more energy efficient one brings future savings in energy expenditures, the initial investment for replacing the existing technologies can be financed from the future savings. Energy Service Companies (ESCOs) use a business model that is based on offering energy saving technologies as a service (Goldman et al. 2005, Vine 2005, Yavuz 2013). In the business model we consider, a firm offers making all the necessary energy saving technology investments and replacements for a customer in exchange of getting a fraction of the savings in energy expenditures for a predetermined time period.

Feasibility of this business model is affected by variation in energy consumption, uncertainty in energy prices and useful life of a technology, and also it is affected by improvement in energy efficiencies and costs of technologies with time, and revenue from carbon offsets.

Our objective in this study is to develop an analytical model to answer the following questions: *How do operating cost, efficiency, useful life, energy price, and energy usage affect which energy saving technology to be chosen to achieve a lower total expected cost of operation; What factors determine the feasibility of offering energy saving technologies as a service?*, and *How can the contract parameters of the business model be set to ensure the feasibility of the business model?* In order to answer these questions, we develop and analyze a stochastic model that captures the main issues related to technology replacement decisions to improve energy efficiency.

Although business models to offer energy saving technologies as a service received considerable attention in the energy literature, it has not received the same attention in production research literature. The existing studies in operations management literature focus on managerial decision making process regarding adopting energy saving technologies. By using a large data set, Aflaki et al. (2013) discuss the decision making process to select and implement energy efficiency measures in manufacturing companies. Muthulingam et al. (2013) discuss the adoption of energy efficiency improvement recommendations by small and medium-sized manufacturing companies. These studies include the effects of the cost considerations among other factors by focusing on the decision making process without using a detailed operational model.

Tan et al. (2015) present a mathematical programming formulation to select energy efficiency measures for existing buildings in a deterministic setting. By using the data for a campus and the

optimal selection methodology in a multi-period setting, they also investigate the operation of a business model to offer energy saving technologies as a service. However, due to using a deterministic model, the effects of uncertainty in energy prices and useful life of the technologies are not captured in this model.

Our study combines a model for energy consumption with a model for the energy price to derive the total cost of operation. In the literature, the studies on modelling energy consumption, and the studies on modelling energy price are developed separately. There are numerous studies in the literature on modeling residential demand for electricity and natural gas. There are different methodologies that are used to estimate both the short-run and the long-run residential demand from the aggregate data (Houthakker 1951, Liu and Lin 1991, Eltony 1996, Smith et al. 1996, Garcia-Cerrutti 2000, Holtedahl and Joutz 2004, Mohammadi 2009). Together with econometrical methodologies determined by price and income elasticity, Heating Day Degrees and Cooling Day Degrees are used in many models to forecast the residential natural gas demand (Sarak and Satman. 2003, Belzer and Cort 2004, Krese et al. 2012).

The price of energy depends on a range of different supply and demand conditions, including the geopolitical situation, import diversification, distribution costs, environmental protection costs, and weather conditions. The general approach for energy price modelling in the literature is to model the logarithmic energy spot prices through a mean-reverting process, such that in the classical Gaussian setting the spot price dynamics become lognormal (Geman and Roncoroni 2006, Benth et al. 2007). The energy price is also commonly modelled as the Ornstein-Uhlenbeck process (Renshaw 1987).

In our model, we combine the Heating Day Degrees approach to model energy consumption with the energy price model that is described as the discrete analogue of the Ornstein-Uhlenbeck process to determine the total energy usage cost.

By using a stochastic model, we first determine the conditions to select a technology that yields a lower expected cost of energy usage and replacement cost compared to another technology. Then we analyze the feasibility of the business model for the service provider by using expected cost and also value-at-risk criteria. We show that when the service provider selects the contract parameters, that determine how the savings will be shared between the service provider and the customer and the duration of the contract, a right way by considering the initial cost of technology, efficiency improvement, and also the risks due to uncertainty in energy prices and useful life, it can gain significant financial benefit. The customer also benefits financially from reduction in energy usage and replacement costs, and also from additional revenue obtained through selling carbon offsets. Furthermore, since this business plan is based on increasing energy efficiency, the proposed approach

decreases energy consumption, and therefore carbon dioxide emissions. As a result, by using an analytical model, we show that offering energy saving technologies as a service is a *win-win-win* situation for the service provider, its customer, and for the environment.

Our model is the only study in the literature that develops a detailed analytical model for the operation of energy saving companies in such a way that captures improvement in energy efficiencies and costs of technologies with time, variation in energy consumption, uncertainty in energy unit price and useful life of a technology, and revenue from carbon offsets simultaneously in a stochastic setting to analyze a business plan to offer energy saving technologies as a service.

We consider developing a detailed stochastic model that captures salient characteristics that affect technology replacement decisions to improve energy efficiency as our first contribution. Furthermore, we consider developing a detailed analytical model for the operation of energy saving companies and showing the conditions for the feasibility of a business plan that is based on offering technology replacement as a service in a stochastic setting as the second contribution of this study.

The organization of the remaining part of this paper is as follows. In Section 2, we present the model for a single technology. By using this model, the technology replacement problem between two technologies is addressed in Section 3. In Section 4, we present a business model to offer energy saving technologies as a service, and analyze its feasibility. In Section 5, we present numerical results that investigate the feasibility of the business model. Finally, the conclusions are given in Section 6.

2. Model

Our modelling objective is deriving an analytical expression for the total cost of operation including the cost of energy usage, the cost of replacements, and the revenue from carbon offsets for a given technology. By comparing the total cost of operation for two different technologies, we can select the one that yields a lower total cost compared to the other one. Since technology selection may also include other considerations beyond the total cost of operation, we only focus on the total cost of operation in this paper. By using the analytical expression for the total cost of operation, we also analyze the feasibility of a business model to offer energy efficiency technologies as a service, and present a method to set the contract parameters of the business model.

In order to introduce our modelling approach, we first focus on a single technology and include important features such as improvements in energy efficiency and cost of technologies, variations in electricity usage and price, random lifetime of technologies, and revenue from carbon offsets that depend on CO_2 emissions. Namely, energy efficiency of the technology improves, and the cost of the technology decreases with time. Therefore, at each replacement, more energy can be obtained per

dollar spent for replacement. Energy usage may also change with time, and future energy prices are random. As a result, the cost of using energy during a period of time depends on the energy efficiency of the technology used, energy usage during this time, and the electricity price. Furthermore when a technology is installed, it has a random lifetime. When it fails, it is replaced with a new one that may be cheaper and also more efficient.

Combining the cost of replacing the technology during a given period τ , denoted with $\pi_r(\tau)$, with the energy usage cost during the same period, denoted with $\pi_e(\tau)$ gives the total cost of replacement and usage. Energy saving also leads to a reduction in CO_2 emissions. The carbon offset that is the amount of CO_2 saved can also be sold at a carbon offset market within an allowable limit set by regulators to bring additional revenue. The revenue from CO_2 sold at the carbon offset markets is denoted with $\pi_c(\tau)$. The difference between the cost of replacement and usage and the revenue from carbon offsets gives the total cost of operation for a given technology during the same period, $\Pi(\tau)$:

$$\Pi(\tau) = \pi_r(\tau) + \pi_e(\tau) - \pi_c(\tau). \quad (1)$$

Due to stochastic nature of the model with uncertain lifetime and uncertain energy prices, the total cost is a random variable. In this section, we start with the model of a single technology and then determine the expressions for the replacement cost, usage cost, and the revenue from carbon offsets depending on improvement in energy efficiencies and costs of technologies with time, variation in energy consumption, uncertainty in energy unit price and useful life of a technology. We then derive the expectation and the variance of the total cost of operation.

2.1. Model of a Single Technology

In our model, we capture the cost of a given technology, its energy conversion efficiency, energy consumption, and the limits for selling CO_2 savings as deterministic functions of time. The random variables in the model are the energy unit price and the useful life of the technology. The time unit is taken as one month. In the analysis of the business model, the system is analyzed for τ months, $T = \tau/12$ years.

2.1.1. Cost of Single Technology We assume that the unit price of a technology decreases exponentially with time. The price at time t is $ce^{-\beta't}$ where c is the initial cost when it is first installed and β' is the rate of cost decrease.

Figure 1(a) and Figure 1(b) show observed values and the fitted exponential cost functions for Compact Fluorescent Light (CFL) and LED prices respectively. Figure 1(a) is based on a study in USA (Houri and Khoury 2010). Figure 1(b) is based on a study on LED prices in Japan (LEDinside 2011).

Figure 1 Price decrease of CFL and LED prices

Figure 2 Technology improvement in lighting industry ($\epsilon = 0.0015$, $\eta = 0.347$, $t = 0 : 1970$)

2.1.2. Efficiency Improvements The energy conversion efficiency of technology is the ratio between the useful output of an energy conversion machine and the input. The energy conversion efficiency of technology at the time of installation is denoted with ϵ . We assume that the energy efficiency improves exponentially with time following the advances in technology. As a result, the energy conversion efficiency of technology at time t , $\epsilon(t)$ is expressed as $\epsilon e^{\eta t}$ where η is the rate of technological improvement.

For example, the energy conversion efficiency of a bulb is expressed as the light produced (measured in lumens) per energy used (measured in watts). Figure 2 shows that the improvement of energy conversion efficiency (lumen/watt value) achieved in years.

2.1.3. Useful Life of a given Technology The useful life of a technology is modelled as an exponentially distributed random variable with rate λ . The end of useful life of a technology is

Figure 3 Unit price of natural gas over time in Turkey and the trend line ($m = 0.004$, $n = 0.262$, $\varphi = 0.860$, $\sigma = 0.004$)

referred as a *failure*. Accordingly, λ is also the *failure rate*.

We assume that a technology is fully operational until the end of its useful life and operates with the energy conversion efficiency that is improving with time. At the end of its useful life (equivalently when it fails), it is replaced with a newer version of the technology that has a lower cost since the cost is also decreasing with time.

2.1.4. Energy Unit Price For our model, the energy price that is set by the utility companies is relevant. The price set by a utility company is less variable for a period of time compared to the spot price. We consider energy expenditures that occur monthly. Accordingly, we model the energy price at period t , $p(t)$ as

$$p(t) = f(t) + X(t) \quad (2)$$

where $f(t)$ is a deterministic function that describes the energy price forecast, and $X(t)$ is an autoregressive process with order 1. This representation is based on the discrete analogue of the Ornstein-Uhlenbeck process that is commonly used for energy price modelling (Renshaw 1987). More specifically,

$$X(t) = \varphi X(t-1) + \vartheta \quad (3)$$

where $\vartheta \sim N(0, \sigma^2)$.

Figure 3 shows the unit price of natural gas in Turkey. In this case, a linear function $f(t) = mt + n$ is used for the deterministic part of $p(t)$ and the parameters φ and σ^2 are estimated from the data.

Figure 4 Observed consumption of natural gas at Koç University and result of the predicted consumption with HDD values ($a = 575.8$, $b = 20044.2$, $h = [363, 341, 315, 171, 65, 3, 0, 0, 6, 79, 213, 309]$)

2.1.5. Energy Consumption Monthly energy consumption at month t is denoted with $\omega(t)$. The monthly energy consumption is modeled as $\omega(t) = ah(t) + b$ where $h(t)$ is Heating Day Degree in month t and a and b are constants determined by using linear regression. Heating Day Degree is defined as the measurement of the demand of energy to heat a building depending on the difference between the inner and outer temperatures in each day of the year (Sarak and Satman. 2003, Belzer and Cort 2004, Krese et al. 2012). Therefore, Heating Day Degree in month t , $h(t)$ is defined as

$$h(t) = \begin{cases} \sum_{j=1}^{D(t)} T_j^i(t) - T_j^o(t), & \text{if } T_j^i(t) > T_j^o(t), \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where $T_j^i(t)$ is the inside temperature on the j^{th} day of month t , $T_j^o(t)$ is the outside temperature on the j^{th} day of month t , and $D(t)$ is the number of days in month t .

Figure 4 shows the natural gas consumption of Koç University from 2004 to 2011, and the prediction of the consumption generated by using the 60-year averages of Heating Day Degrees in Istanbul. Our regression analysis showed that the best performing forecasts are obtained by using the 60-year averages of Heating Day Degrees compared to using averages of more recent years (Yavuz 2013). The small declines observed for the winter months in the consumption data for Koç University are due to the semester breaks in the academic calendar of the university. This model assumes that the annual energy usage stays the same from one year to another year (see Figure 4 as an example). If there is an increase expected in annual consumption, another term can be added to the model to capture the increase.

2.1.6. Revenue from Carbon Offset Markets Our model incorporates possible additional revenue from sales of reduced carbon dioxide emissions at the carbon offset markets. Since replacing the existing technology with a more efficient one also reduces the carbon dioxide emissions, this brings additional financial benefit to the company that ease the financial burden of technology replacement.

A carbon offset is a reduction in emissions of carbon dioxide or greenhouse gases made in order to compensate for an emission made elsewhere. According to the Kyoto Protocol, a firm can sell the saved amount of CO_2 at a carbon offset market within the allowable limits. In our model, we include the earnings a firm obtains by selling the reduction in CO_2 emissions as a result of replacing its existing technology with a more efficient one. The amount of CO_2 produced from the usage of 1 kWh of energy is denoted with θ , and the price per kg of CO_2 sold at the carbon offset market is assumed to be constant with value p_c . We assume that in each period, a company is allowed to sell the difference between an upper limit of L kg of CO_2 that is set by the regulators and the actual emission. This limit is expected to decrease with time as a result of the decrease of the incentives and improvements in technology. In other words, the regulators give more financial incentive to actions that will reduce carbon dioxide emissions now. However, operating with lower carbon dioxide emission will be a requirement as the time goes on. So the financial incentives will decrease with time. As a result, this limit is expected to decrease with a rate of ζ every year. That is, the maximum amount that can be sold at a carbon offset market at time t is $Le^{-\zeta t}$.

2.2. Total Cost for a Single Technology

Once the model parameters are determined, the total cost that is the difference between the cost of replacement and energy usage and the revenue from the carbon offset markets in τ months can be determined.

2.2.1. Cost of Replacement After a technology is installed at a cost of c at time 0, it may need to be replaced multiple times due to reaching the end of its useful life, i.e., due to a *failure* before the end of the planning horizon τ .

For a realization, we denote the time of the n^{th} failure as T_n . The replacements take place immediately at time instances T_n at a cost of $ce^{-\beta' T_n}$, $n = 1, 2, \dots$, $T_n \leq \tau$. If the interest rate per period is i , the present value of the cost of all replacements during $[0, \tau]$ denoted with $\pi_r(\tau)$, is given as

$$\pi_r(\tau) = \sum_{n=0}^{\infty} (1+i)^{-T_n} ce^{-\beta' T_n} 1_{\{T_n \leq \tau\}} = \sum_{n=0}^{\infty} c((1+i)e^{\beta'})^{-T_n} 1_{\{T_n \leq \tau\}}. \quad (5)$$

Since the useful life of a technology and also the electricity price are random variables, $\pi_r(\tau)$ is also a random variable. The expectation and the variance of the energy usage cost $E[\pi_r(\tau)]$ and

$Var[\pi_r(\tau)]$ are given in the Appendix.

2.2.2. Cost of Energy Usage The cost of energy consumption depends on how much input energy is used and also the price of energy at time t . Since the energy conversion efficiency at time t is $\varepsilon(t)$ and the energy consumption is $\omega(t)$, the amount of energy that will be used to satisfy the demand at time t is $\frac{\omega(t)}{\varepsilon(t)}$. Then the net present value of the cost of energy consumption during $[0, \tau]$ $\pi_e(\tau)$ is

$$\pi_e(\tau) = \sum_t (1+i)^{-t} p(t) \frac{\omega(t)}{\varepsilon(t)} = \sum_t ((1+i)e^\eta)^{-t} p(t) \frac{\omega(t)}{\varepsilon}. \quad (6)$$

Let us define a new discount rate r as $((1+i)e^\eta) = (1+r)$ and $\alpha = \ln(1+r)$. Furthermore, let us define $e^{\beta' - \eta} = e^\beta$. With these change of variables, Equations (5) and (6) are rewritten as

$$\pi_r(\tau) = \sum_{n=0}^{\infty} c e^{-(\alpha+\beta)T_n} 1_{\{T_n \leq \tau\}} \quad (7)$$

and

$$\pi_e(\tau) = \sum_t e^{-\alpha t} p(t) \frac{\omega(t)}{\varepsilon}. \quad (8)$$

In other words, the effect of cost decrease and the effect of energy conversion efficiency improvement on the cost can be captured by redefining the rate of decrease and the interest rate. In the remaining part of the paper, the energy conversion efficiency is taken as constant without loss of generality with this transformation.

Since the useful life of a technology and also the electricity price are random variables, $\pi_e(\tau)$ is also a random variable. In the Appendix, the expectation and the variance of the energy usage cost, $E[\pi_e(\tau)]$ and $Var[\pi_e(\tau)]$ are derived for the case when the energy unit price is modelled by using a linear term and an autoregressive error term: $p(t) = mt + n + X(t)$, $X(t) = \varphi X(t-1) + \vartheta$ where $\vartheta \sim N(0, \sigma^2)$, and the energy consumption is represented as $\omega(t) = ah(t) + b$ where $h(t)$ is the number of heating degree days in each month that is a periodic function with a period of 12.

2.2.3. Total CO_2 Emission and Revenue from Carbon Offsets The total CO_2 produced during τ months is based on the total energy used during the same period. The total CO_2 emission, denoted with $q_c(\tau)$ is determined as

$$q_c(\tau) = \frac{\tau\theta}{12\varepsilon} \sum_{t=1}^{12} \omega(t) \quad (9)$$

where θ is the amount of CO_2 emission generated by consuming a given level of energy,

The net present value of the revenue from CO_2 sales, $\pi_c(\tau)$ is calculated as the difference between the revenue of the maximum allowable amount of CO_2 sold at carbon offset market and the lost revenue due to total emission that results from the energy consumption during $[0, \tau]$:

$$\pi_c(\tau) = \sum_{t=0}^{\tau} (1+i)^{-t} p_c L e^{-\zeta t} - \sum_{t=1}^{12} \sum_{t'=1}^{\tau/12} (1+r)^{-12(t'-1)-t} \frac{\omega(t)}{\varepsilon} p_c \theta.$$

2.2.4. Total Cost of Operation Combining the expected cost of replacement, energy usage, and revenue from CO_2 sales, the total expected cost of using a given technology in τ periods $E[\Pi(\tau)]$ is given as

$$E[\Pi(\tau)] = E[\pi_r(\tau)] + E[\pi_e(\tau)] - \pi_c(\tau). \quad (10)$$

Since the uncertainty in the useful life of a technology does not affect the electricity prices, $\pi_r(\tau)$ and $\pi_e(\tau)$ are independent, and the revenue from CO_2 sales is constant. Then

$$Var[\Pi(\tau)] = Var[\pi_r(\tau)] + Var[\pi_e(\tau)] \quad (11)$$

which can be approximated for large τ as

$$Var[\Pi(\tau)] \cong Var[\pi_r] + Var[\pi_e(\tau)] \quad (12)$$

where $Var[\pi_r]$ is the asymptotic variance of the replacement cost. $E[\pi_r(\tau)]$, $E[\pi_e(\tau)]$, $Var[\pi_r]$, and $Var[\pi_e(\tau)]$ that appear in the above equations can be calculated by using the expressions given in the Appendix for given values of the system parameters.

Since $\Pi(\tau)$ is the sum of costs at each period, $\Pi(\tau)$ can be approximated as a normal distribution with mean $E[\Pi(\tau)]$ given in Equation (10) and variance $Var[\Pi(\tau)]$ given in Equation (12) for long contract durations following the central limit theorem. Accordingly, we can use the asymptotic distribution of the total cost of operation to analyze the feasibility of the business model.

Next we will use the total cost of operation to determine under which conditions a particular technology is preferred to another one from the total cost of operation perspective. Then we will analyze feasibility of offering energy saving technologies as a service under risk considerations.

3. Replacing an Existing Technology with a More Efficient Technology

In Section 2, the total cost of operating a given technology is determined by using the cost of the technology, its energy conversion efficiency, energy consumption, useful life of technology, electricity unit price, and revenue from carbon offset. Comparing the total cost of two technologies with different

energy conversion efficiencies, costs, and failure rates during a given time period can be used to determine which technology is a better choice. We first present the conditions that yield choosing one technology over another one to achieve the lowest expected total cost.

We consider two technologies Technology 1 and Technology 2 with different initial costs c_1 and c_2 , initial efficiencies ε_1 and ε_2 , and useful lives with rates λ_1 and λ_2 . We assume that the rate of decrease of costs and the rate of increase of efficiencies are the same for both technologies. These two technologies are being evaluated for τ periods in order to determine the technology that will yield the lowest expected total cost of operation.

Our main result summarizes the conditions when one technology is preferred to another one.

Proposition 1 *Let c_1 and c_2 be the unit cost of technologies and ε_1 and ε_2 are the initial efficiencies for technology 1 and technology 2, respectively. The table shows which technology should be selected as a result of given comparisons*

	$\varepsilon_1 > \varepsilon_2$	$\varepsilon_1 = \varepsilon_2$	$\varepsilon_1 < \varepsilon_2$
$c_1\mu_1(\tau) < c_2\mu_2(\tau)$	1	1	$\frac{(c_1\mu_1(\tau) - c_2\mu_2(\tau))\varepsilon_1\varepsilon_2}{\varepsilon_1 - \varepsilon_2} > \xi \Rightarrow 2$
$c_1\mu_1(\tau) = c_2\mu_2(\tau)$	1	1 or 2	2
$c_1\mu_1(\tau) > c_2\mu_2(\tau)$	$\frac{(c_1\mu_1(\tau) - c_2\mu_2(\tau))\varepsilon_1\varepsilon_2}{\varepsilon_1 - \varepsilon_2} > \xi \Rightarrow 2$	2	2

where $\mu_1(\tau) = \lambda_1 p(\tau) + \alpha + \beta$, $\mu_2(\tau) = \lambda_2 p(\tau) + \alpha + \beta$ and $\xi = (\alpha + \beta)(R\widehat{M}\widehat{A} - \pi\theta R\widehat{1}\widehat{A})$ (defined in the Appendix), and $\widehat{1}$ is a $\tau \times 12$ matrix of ones.

Proof This proposition follows directly determining the conditions that make the difference between the expected total cost of operation for Technology 1 and Technology 2, obtained from Equation (10) greater than 0, equal to 0, or less than 0. The proof is algebraic and omitted here. $\square$

The result shows that if a technology is more efficient and also has a lower cost to operate, it should be selected. If the operating costs are the same, the one with higher efficiency should be selected. Similarly, if the efficiencies are the same, the one with the lower operating cost should be selected. If both the costs and the efficiencies are the same, we are indifferent between choosing either of the technologies. Otherwise, that is if one technology is cheaper to operate and the other one is more efficient, a condition that involves the expected total costs should be checked with a threshold to determine which technology to select.

4. Analysis of a Business Model to Offer Energy Saving Technologies as a Service

We now focus on a business plan to offer an energy saving technology as a service. In this arrangement, the existing technology is replaced with a more efficient one. The total differential cost of replacing an existing technology with a more efficient one is financed from the future energy savings that are shared between the service provider that installs the more efficient technology and its customer.

Depending on the terms of the contract that are the energy saving ratio that will be guaranteed to the client and the length of the contract, the parameters that describe the technologies that are the energy conversion efficiencies of the existing and the new technologies and the failure rate of the new technology, the initial and operating costs of installing a new technology, and the realization of the uncertain price and usage, this business plan brings a profit or loss to the service provider.

Since the business plan operates in a random environment, we discuss the feasibility of the business plan and determine its contract parameters according to the expected cost and value-at-risk criteria. Moreover, we discuss the risks faced by the service provider due to uncertainty in energy prices and random life of technologies, and show how the parameters can be set to mitigate these risks.

4.1. Contract between the Service Provider and the Client

According to the business model, a service provider offers a contract (Δ, τ) where Δ is the fraction of the current energy usage cost that the client pays after the instalment of the new technology and referred as the reduction rate, and τ is the length of the time this contract will be valid. The service provider is also responsible for replacements of the new technology in case the useful life of the technology is shorter, or the technology fails during the contract period.

The service provider makes an initial investment of c_0 to improve the energy conversion efficiency of the client from its current level of ε to ε' where $\varepsilon' > \varepsilon$. We assume that the rate of decrease of the cost of the technology and the rate of the increase of the technology are the same for both existing and the new technologies. The failure rate of the new technology is λ . According to the contract, the service provider receives $1 - \Delta$ fraction of the existing energy usage cost of the client as a payment for τ periods. The total operating cost of the service provider for the contract period is denoted with κ .

For example, the contract $(0.8, 5)$ indicates that the service provider guarantees a saving of 20% to the client compared to its current energy cost for a period of 5 years. With this contract, the client pays 80% of what it is currently paying to the service provider for 5 years. If the service provider makes the necessary investments in energy saving technologies and obtain a cost saving greater than 20%, the difference between what it collects from the customer and what it pays as the energy cost

can finance its initial investment, cover its operational cost, and bring profit.

For all $\Delta < 1$, the client decreases its energy usage cost, and it also benefits from not incurring the future replacement costs of the technology used. Furthermore since using a more efficient technology also decreases the CO_2 emissions, there will be an additional financial benefit from increased revenue from carbon offset. In the business model, this additional revenue is kept by the customer. As a result, this arrangement is always beneficial for the customer.

However, depending on the terms of the contract (Δ, τ) , the efficiencies of the current and the new technologies $(\varepsilon, \varepsilon')$, the costs (c_0, κ) , electricity usage in each period $\omega(t)$, and the realization of the uncertain price $p(t)$ and the number of necessary replacements due to failures with rate λ , this business plan brings a profit or loss to the service provider.

We use the model described in Section 2 to determine when this business model brings profits and how the parameters of the contract (Δ, τ) should be set according to different criteria. Since the business plan operates in a random environment, we use both an expectation criterion and also a probabilistic criterion to set the contract parameters.

4.2. Feasibility of the Business Plan

We denote the energy usage cost and the revenue obtained by selling CO_2 by using a technology with an efficiency of ε in a period of τ time units as $\pi_e(\tau, \varepsilon)$ and $\pi_c(\tau, \varepsilon)$.

For this business model to be feasible for the service provider, the difference between the current energy usage cost and the energy usage cost with the new technology should be high enough to cover the cost of replacement during the contract period and also the operating cost of the service provider. Even if all the reduction in energy expenditures are kept by the service provider, i.e., the client receives no reduction ($\Delta = 1$) and pays the same bill, the following inequality must hold:

$$\pi_e(\tau, \varepsilon) - \pi_e(\tau, \varepsilon') - \pi_r(\tau) > \kappa. \quad (13)$$

Let the profit of the customer, that is the financial benefit of the customer agreeing to this service be $\Pi_A(\Delta, \tau)$, and the profit of the service provider be $\Pi_S(\Delta, \tau)$. Considering the reduction rate offered to the customer, $\Pi_A(\Delta, \tau)$ and $\Pi_S(\Delta, \tau)$ can be expressed as

$$\Pi_A(\Delta, \tau) = (1 - \Delta)\pi_e(\tau, \varepsilon) + \pi_c(\tau, \varepsilon') - \pi_c(\tau, \varepsilon) + \pi_r(\tau) \quad (14)$$

$$\Pi_S(\Delta, \tau) = \Delta\pi_e(\tau, \varepsilon) - \pi_e(\tau, \varepsilon') - \pi_r(\tau). \quad (15)$$

Equation (14) shows that the profit of the customer is the difference between what it used to pay and what it pays now with the contract and the differential gain of CO_2 offset sales due to increased

emission savings with the new technology. Similarly, the profit of the service provider is the difference between what it collects from the customer based on the contract and what it pays to the utility company for the energy usage of the customer and the cost of replacements during the contract period as shown in Equation (15).

Then the total financial benefit created by this service is shared between the customer and the service provider as

$$\pi_e(\tau, \varepsilon) - \pi_e(\tau, \varepsilon') + \pi_e(\tau, \varepsilon') - \pi_e(\tau, \varepsilon) = \Pi_A(\Delta, \tau) + \Pi_S(\Delta, \tau). \quad (16)$$

Note that Δ does not appear in the expression for the total benefit since the reduction rate Δ determines how the total benefit $\Pi_A(\Delta, \tau) + \Pi_S(\Delta, \tau)$ is shared between the customer and the service provider. If $\Delta = 1$, then the customer does not benefit from the reduction in energy expenditures. Its main benefit will be the increase revenue from CO_2 sales and the replacement cost that will be covered by the service provider. In this case, the service provider keeps all the benefit due to decreasing energy cost. However, if $\Delta = \Delta^*$, then the customer makes the highest profit, and Δ^* is the maximum reduction that the service provider can offer to its client and still maintains a non-negative profit that is high enough to cover the operating cost. We determine Δ^* by comparing $\Pi_S(\Delta^*, \tau)$ with κ .

Similarly $\pi_r(\tau)$ does not appear in the expression for the total profit since it stays the same and the business plan allocates this cost to the service provider instead of the customer.

4.3. Setting the Contract Terms with the Expected Cost Criterion

When the expected cost is used as the main criterion to analyze the business model, the overall feasibility condition is given by using Equation (13) as

$$E[\pi_e(\tau, \varepsilon)] - E[\pi_e(\tau, \varepsilon')] - E[\pi_r(\tau)] > \kappa. \quad (17)$$

By using Equations (20) and (21) given in the Appendix, the feasibility condition for the service provider is given as

$$\frac{1}{\varepsilon} - \frac{1}{\varepsilon'} > \frac{\frac{c_0 \lambda}{\alpha + \beta} (1 - e^{-(\alpha + \beta)\tau}) + c_0 + \kappa}{R \widehat{M} \widehat{A}}.$$

where R , $\widehat{M}$, and $\widehat{A}$ are given in the Appendix. Furthermore, we can determine the maximum reduction rate the service provider can offer by determining the discount rate that ensures $E[\Pi_S] \geq \kappa$, or equivalently,

$$\left(\frac{1}{\varepsilon} - \frac{\Delta}{\varepsilon} - \frac{1}{\varepsilon'} \right) R \widehat{M} \widehat{A} - \frac{c_0 \lambda}{\alpha + \beta} (1 - e^{-(\alpha + \beta)\tau}) - c_0 \geq \kappa$$

Then the maximum discount that can be offered to the customer

$$\Delta^* = -\frac{\kappa + \frac{c_0\lambda}{\alpha+\beta}(1 - e^{-(\alpha+\beta)\tau}) + c_0}{R\widehat{M}\widehat{A}} + 1 - \frac{\varepsilon}{\varepsilon'}.$$

Finally, the reduction rate the service provider can offer should satisfy $\Delta^* < \Delta < 1$ in order for this business model to be feasible for the service provider according to the expected cost criterion.

For a given reduction rate, the above inequality also gives the maximum initial investment that a service provider can afford to invest in a new technology to ensure profitability:

$$c_0 \leq \frac{(\frac{1}{\varepsilon} - \frac{\Delta}{\varepsilon} - \frac{1}{\varepsilon'})R\widehat{M}\widehat{A}}{\frac{(1 - e^{-(\alpha+\beta)\tau})\lambda}{\alpha+\beta} + 1}.$$

4.4. Setting the Contract Terms Under Risk Considerations

In the preceding section, we analyzed the feasibility of the business model by taking the expected cost criterion as the main criterion. However, since the energy price and the useful life of the projects are random, the expected cost criterion may impose significant risks for the service provider. In this section, we incorporate the effects of uncertainty imposed by uncertain energy prices and useful life of technologies on the net present value of the profit of the service provider to analyze the business model. We focus on determining the reduction rate by taking Value-at-Risk considerations, and also focus on the risk of loss for the service provider.

4.4.1. Value-at-Risk Consideration One way of managing the risks is selecting the maximum discount rate to ensure that the probability of making a profit that is higher than a given desired level ψ is at least a given level ρ , i.e.

$$P\{\Pi_S(\Delta, \tau) > \psi\} > \rho. \quad (18)$$

Since the profit of the service provider is the sum of monthly or annual payments received from the energy usage costs of its client, the distribution of Π_S is approximately normal following the central limit theorem. Under the normality assumption of the profit of the service provider, the condition $P\{\Pi_S > \psi\} > \rho$ can be rewritten as

$$E[\Pi_S(\Delta, \tau)] > \psi - \phi^{-1}(1 - \rho)\sigma_{\Pi_S}$$

where σ_{Π_S} is the standard deviation of the profit of the service provider and ϕ^{-1} is the inverse of the normal distribution function.

Now the feasibility condition for the business plan with the Value-at-Risk consideration can be written as

$$\left(\frac{1}{\varepsilon} - \frac{\Delta}{\varepsilon} - \frac{1}{\varepsilon'}\right)R\widehat{M}\widehat{A} - \frac{c_0\lambda}{\alpha+\beta}(1 - e^{-(\alpha+\beta)\tau}) - c_0 > \psi - \phi^{-1}(1 - \rho)\sigma_{\Pi_S}.$$

where $\sigma_{\Pi_S} = \sqrt{\text{Var}[\Pi_S(\Delta, \tau)]}$. The variance of the net present value of the profit of the service provider $\text{Var}[\Pi_S(\Delta, \tau)]$ can be determined from Equation (15).

The above inequality must hold for the first feasibility condition when $\Delta = 0$ to ensure overall feasibility:

$$\left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon'}\right) R\widehat{M}\widehat{A} - \frac{c_0\lambda}{\alpha + \beta}(1 - e^{-(\alpha+\beta)\tau}) - c > \psi - \phi^{-1}(1 - \rho)\sigma_{\Pi_S}.$$

The maximum discount rate Δ^* that can be offered with the Value-at-Risk consideration can also be determined from the above inequality as

$$\Delta^* = \frac{\left(\psi - \phi^{-1}(1 - \rho)\sigma_{\Pi_S} + \frac{c_0\lambda}{\alpha + \beta}(1 - e^{-(\alpha+\beta)\tau}) + c_0\right)\varepsilon}{R\widehat{M}\widehat{A}} + \frac{\varepsilon}{\varepsilon'} - 1.$$

As a result, the discount rate the service provider can offer should satisfy $\Delta^* < \Delta < 1$ in order for the service provider to guarantee that the probability of achieving a profit level of at least ψ is greater than ρ .

4.4.2. Probability of Loss From the service provider's point of view, uncertainties in the energy price and the useful life of the technologies impose a risk. The most important risk is the risk of loss, i.e., risk of not being able to cover the operational cost and the replacement costs from the share of the benefit from the energy expenditure savings and incurring a loss.

Since the distribution of the profit of the service provider is normal for long contract periods, the probability of loss $P\{\Pi_S < 0\}$ can be evaluated as

$$P\{\Pi_S(\Delta, \tau) < 0\} = \phi\left(\frac{-E[\Pi_S(\Delta, \tau)]}{\sigma_{\Pi_S}}\right). \quad (19)$$

where $\phi(\cdot)$ is the cumulative distribution function of standard normal distribution.

The analytical model described in Sections 2 and 4 captures the salient characteristics of a business model to offer energy saving technologies as a service. In the next section, we examine the model numerically.

5. Numerical Results

In order to analyze a business plan to offer energy saving technologies as a service by using the model described in the preceding sections numerically, we focus on three sets of experiments. In all the experiments, the numerical values of the parameters are set by using the data presented in Section 2.1.

The stochastic model presented in this study yields closed-form expressions for the expectation and variance of different cost terms. Although these expressions are lengthy, calculation time is

almost instantaneous, and depends only on the number of operations to calculate the expressions. The numerical results are obtained by using Matlab.

First, we analyze the effect of the initial investment and the minimum reduction rate on the feasibility of the business plan. Second, we analyze the effect of contract parameters and the difference between the efficiencies of the old and the new technologies on the probability of loss. Finally, we analyze the effects of uncertainties in the energy price and in the useful life of the technologies on the results.

5.1. Effect of the Initial Investment and the Minimum Reduction Rate on the Feasibility of the Business Plan

Figures 5(a), 5(b) and 5(c) show the effect of initial investment on the minimum reduction rate that can be given to the customers in order to have a feasible operation of the service provider. These figures show that as initial investment of a technology increases, the minimum reduction rate that a company can offer decreases. Figure 5(a) shows that there is no difference in rates of the decrease for different technologies when $\varepsilon/\varepsilon' = 0.2$ or $\varepsilon/\varepsilon' = 0.5$. However, Figure 5(b) illustrates that the rates of decrease depends on the length of the contract used, since longer contracts allow the service provider to make higher investments. Figure 5(c) is an another way of capturing the relationship depicted in Figure 5(a) by comparing the proportion of initial investments to the net present value of the invested technology on the discount rate.

5.2. Effect of Contract Parameters and Technology on the Probability of Loss

Figure 6(a) and 6(b) show the changes in the probability of loss as the reduction rate Δ and the ratio of the efficiency of old technology to the new technology ε/ε' increases, respectively.

As expected, Figure 6(a) indicates that giving higher reductions can cause a loss. Figure 6(b) indicates that investing in technologies that do not improve the efficiency significantly also increases the probability of loss. Choosing more efficient technologies (lower ε/ε') is more appropriate to avoid a loss.

5.3. Comparing Uncertainty in the Model

In this section, we analyze the effects of random variables of the model that are replacement periods and energy price variation to determine which random variables affects the variability most. In the experiments, we vary the parameters of these random variables, and also set them as deterministic variables and report their effect on the variance and the coefficient of variation of the profit of the service provider.

Figure 7(a) and 7(b) show the effect of price uncertainty and failure rate on the variability of the

(a) Effect of efficiency improvement ratio ($\tau = 25$)(b) Effect of contract length ($\epsilon/\epsilon' = 0.2$)

(c) Effect of proportion of initial investment to energy usage cost

($\tau = 25$)**Figure 5 The Initial Investment and the Minimum Reduction Rate** ($m = 0,003$, $n = 0.300$, $G = 1314533$)

profit of service provider. The figure shows that the variability in energy prices affects the variability of the profit more compared to the effect of the variability in the useful life of technologies.

Figures 8 and 9 depict the variance and coefficient of variation of the profit of the service provider, $Var[\Pi_S]$ and $cv[\Pi_S]$, for two cases: when the useful life of the technology is exponential with a given rate and when the useful life is deterministic. Figures show that uncertainty of the useful life does not affect the variation of model in a significant way.

We conduct a similar analysis to analyze the effect of the variability in the energy prices. Figures 10 and 11 show the variance and coefficient of variation of the profit of the service provider when the energy price is modelled as a random variable ($f(t) + X(t)$) and when it is taken as a deterministic function of time ($f(t)$). Figures show that uncertainty of the energy prices affects the variation of

Figure 6 Effect of the Reduction Rate and the Ratio of the Efficiency of Old technology to the New Technology on the Probability of Loss ($r = 0.07$, $c = 20000$, $\lambda = 0.5$, $\varphi = 0.86$, $\sigma = 0.004$, $m = 0.0488$, $n = 0.2781$, $G = 1314533$, $\epsilon = 0.7$, $\tau = 15$)

Figure 7 Effect of Price Uncertainty and Failure Rate on the Variability of the Profit of Service Provider ($r = 0.07$, $c = 10000$, $\varphi = 0.7$, $G = 1342316$, $\epsilon = 0.7$, $\tau = 35$)

model significantly compared to the limited effect of the uncertainty of the useful life.

6. Conclusions

In this study, we model and analyze a business plan to offer energy saving technologies as a service. The important features captured by the model include improvement in energy efficiencies and costs of technologies with time, variation in energy consumption, uncertainty in energy unit price and useful life of a technology, and revenue from carbon offsets.

Figure 8 Comparison of variances in the models when the useful life is exponential and when the useful life is deterministic ($r = 0.07$, $c = 10000$, $\varphi = 0.7$, $G = 1342316$, $\epsilon = 0.7$, $\tau = 35$)

Figure 9 Comparison of CVs in the models when the useful life is exponential and when the useful life is deterministic ($r = 0.07$, $c = 10000$, $\varphi = 0.7$, $G = 1342316$, $\epsilon = 0.7$, $\tau = 35$)

By using the analytical model, we first determine the conditions to select a technology that yields a lower expected cost of energy usage and replacement cost compared to another technology. Then we analyze the feasibility of the business model from the service provider by using expected cost and also Value-at-Risk criteria.

We show that when the service provider selects a technology with higher energy efficiency compared to the existing one and makes an initial investment that is consistent with the saving rate promised to its customer and also the contract duration, the service provider can make financial benefits.

Figure 10 Comparison of variances in the models when σ is constant and the energy price is deterministic ($r = 0.07$, $c = 10000$, $\varphi = 0.7$, $G = 1342316$, $\epsilon = 0.7$, $\tau = 35$)

Figure 11 Comparison of CVs in the models when σ is constant and replacement is deterministic ($r = 0.07$, $c = 10000$, $\varphi = 0.7$, $G = 1342316$, $\epsilon = 0.7$, $\tau = 35$)

The risks mainly due to the uncertainty in energy prices can also be mitigated by selecting contract parameters accordingly. When energy saving technologies are offered as a service, the customer also benefits from savings in energy expenditures, from transferring replacement costs of the technology at the end of useful life to the service provider, and also from additional revenue obtained from increased revenue from carbon offsets. Furthermore, since this business plan is based on increasing the energy efficiency, energy consumption and therefore CO_2 emissions will be decreased. As a result, we show that offering energy saving technologies as a service is a *win-win-win* situation for the service

provider, its customer, and for the environment.

This business model may attract service providers that will undertake technology replacement projects to improve energy efficiency on behalf of manufacturers. As a result, energy efficiency improvement and CO_2 emission reduction initiatives can be accelerated. Furthermore, this business model generates a group of service providers that specialize in technology replacement projects that will save energy. Since these service providers will undertake similar projects for different manufacturers, they will also provide their expertise as an additional benefit.

This model can be extended in a number of ways. The proposed business plan eliminates the risk completely for the customer. As an extension, different business plans can be developed to make the service provider and the customer share the risk differently. Energy price can be modelled by using different approaches. The energy usage can be extended to include annual variations. The number of technologies that can be replaced can also be increased. These extensions are left for future research.

Appendix

A1. Expectation and Variance of the Net Present Value of the Replacement Cost

We first focus on the expected cost of replacement. Since the useful life is an exponential random variable, the time until the n th replacement T_n is an Erlang random variable with n stages. Accordingly, we can determine the expected cost of replacement in τ periods, $E[\pi_r(\tau)]$ as

$$\begin{aligned} E[\pi_r(\tau)] &= c \sum_{n=0}^{\infty} E[e^{-(\alpha+\beta)T_n} 1_{\{T_n \leq \tau\}}] = c \sum_{n=1}^{\infty} \int_0^{\tau} e^{-(\alpha+\beta)s} \frac{\lambda e^{-\lambda s} (\lambda s)^{n-1}}{(n-1)!} ds + c \\ &= \frac{c\lambda}{\alpha + \beta} (1 - e^{-(\alpha+\beta)\tau}) + c. \end{aligned} \quad (20)$$

We use the limiting variance of the replacement cost to approximate the variance of the replacement cost for large τ

$$Var[\pi_r] = E[\pi_r^2] - E^2[\pi_r].$$

The limiting expectation of the replacement cost $E[\pi_r]$ is given as

$$E[\pi_r] = c \sum_{n=0}^{\infty} E[e^{-(\alpha+\beta)T_n}].$$

Note that $E[e^{-(\alpha+\beta)T_n}]$ is the Laplace transform of $\text{Erlang}(n, \lambda)$. Then,

$$E[\pi_r] = c \sum_{n=0}^{\infty} E[e^{-(\alpha+\beta)T_n}] = c \sum_{n=0}^{\infty} \left(\frac{\lambda}{\lambda + \alpha + \beta} \right)^n = \frac{c\lambda}{\alpha + \beta} + c.$$

Now, we determine $E[\pi_r^2]$.

$$E[\pi_r^2] = E \left[c^2 \left(\sum_{n=1}^{\infty} X_n \right)^2 \right] = c^2 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} E[X_n X_m]$$

where $X_n = e^{-(\alpha+\beta)T_n} 1_{\{T_n \leq t\}}$. We will consider three situations $n = m, n < m, n > m$

1-) $n = m$;

$$E[\pi_r^2] = \sum_{n=1}^{\infty} c^2 E[e^{-2(\alpha+\beta)T_n}] = \sum_{n=1}^{\infty} c^2 \left(\frac{\lambda}{\lambda + 2(\alpha + \beta)} \right)^n$$

2-) $n < m$;

$$\begin{aligned} E[\pi_r^2] &= \sum_{n=1}^{\infty} \sum_{m=n+1}^{\infty} c^2 E[X_n X_m] \\ &= \sum_{n=1}^{\infty} \sum_{m=n+1}^{\infty} c^2 E[e^{-(\alpha+\beta)T_n} e^{-(\alpha+\beta)T_m}] \\ &= \sum_{n=1}^{\infty} \sum_{m=n+1}^{\infty} c^2 \left(\frac{\lambda}{\lambda + 2(\alpha + \beta)} \right)^n \left(\frac{\lambda}{\lambda + \alpha + \beta} \right)^{m-n} \end{aligned}$$

3-) $n > m$;

$$\begin{aligned} E[\pi_r^2] &= \sum_{m=1}^{\infty} \sum_{n=m+1}^{\infty} c^2 E[X_n X_m] \\ &= \sum_{m=1}^{\infty} \sum_{n=m+1}^{\infty} c^2 E[e^{-(\alpha+\beta)T_n} e^{-(\alpha+\beta)T_m}] \\ &= \sum_{m=1}^{\infty} \sum_{n=m+1}^{\infty} c^2 \left(\frac{\lambda}{\lambda + 2(\alpha + \beta)} \right)^m \left(\frac{\lambda}{\lambda + \alpha + \beta} \right)^{n-m} \end{aligned}$$

Finally, the variance of the present value of the replacement cost for large τ is

$$\begin{aligned} Var[\pi_r] &= \sum_{n=1}^{\infty} c^2 \left(\frac{\lambda}{\lambda + 2(\alpha + \beta)} \right)^n + \sum_{n=1}^{\infty} \sum_{m=n+1}^{\infty} c^2 \left(\frac{\lambda}{\lambda + 2(\alpha + \beta)} \right)^n \left(\frac{\lambda}{\lambda + \alpha + \beta} \right)^{m-n} \\ &\quad + \sum_{m=1}^{\infty} \sum_{n=m+1}^{\infty} c^2 \left(\frac{\lambda}{\lambda + 2(\alpha + \beta)} \right)^m \left(\frac{\lambda}{\lambda + \alpha + \beta} \right)^{n-m} - \frac{c^2 \lambda^2}{(\alpha + \beta)^2}. \end{aligned}$$

As a result, the variance of the replacement cost for large τ is given as

$$Var[\pi_r] = \frac{c^2 \lambda}{2(\alpha + \beta)}.$$

A2. Expectation and Variance of the Net Present Value of Energy Usage Cost

We now derive the expected cost of energy usage when the energy unit price is modelled by using a linear term and an autoregressive error term: $p(t) = mt + n + X(t)$, $X(t) = \varphi X(t-1) + \vartheta$ where $\vartheta \sim N(0, \sigma^2)$, and the energy consumption is represented as $ah(t) + b$ where $h(t)$ is the number

of heating degree days in each month that is a periodic function with a period of 12. We calculate the total cost of energy usage based on monthly energy bill payments as

$$\begin{aligned} E[\pi_e(\tau)] &= \sum_{t=1}^{12} \sum_{t'=1}^{\tau/12} e^{-12\alpha(t'-1)-t} \frac{\omega(t)}{\varepsilon} (m(12(t'-1) + t) + n + E[X_{12(t'-1)+t}]) \\ &= \sum_{t=1}^{12} e^{-\alpha t} \frac{\omega(t)}{\varepsilon} (mt + n - 12c) \sum_{t'=1}^{\tau/12} e^{-12\alpha(t'-1)} + \sum_{t=1}^{12} e^{-\alpha t} \frac{\omega(t)}{\varepsilon} \sum_{t'=1}^{\tau/12} e^{-12\alpha(t'-1)} 12mt'. \end{aligned}$$

The above equation can be expressed in a more compact form by using a matrix representation:

$$E[\pi_e(\tau)] = \frac{R\widehat{M}\widehat{A}}{\varepsilon} \quad (21)$$

where $R = \{r_i\}$ is $1 \times \tau$ discount vector where $r_i = (1+r)^{-(i-1)}$, $\widehat{M} = \{\widehat{m}_{i,j}\}$ is a $\tau \times 12$ matrix where $\widehat{m}_{i,j} = \widehat{m}_{i,j-1} + m$, $\widehat{m}_{i,1} = \widehat{m}_{i-1,12} + m$, $t \neq 1$ and $\widehat{m}_{1,1} = m + n$, and $\widehat{A} = \{\widehat{d}_j\}$ is a 12×1 vector where $\widehat{d}_j = aW_j + b$.

We now consider the case where the energy usage cost is paid annually. This is based on the annual contract between a service provider who offers energy saving technologies as a service to a customer in return of getting a fraction of energy usage cost calculated annually based on total usage during the year and the current energy price at the time of determination.

Since the annual consumption is taken as the same for every year in period, the annual consumption $G = \sum_{t=1}^{12} \frac{\omega(t)}{\varepsilon}$ is a constant variable. Then, the expected energy usage cost in $T = 12\tau$ years based on annual payments is

$$E[\pi_e(T)] = \sum_{t=0}^T (1+r')^{-t} G(12mt + n). \quad (22)$$

where r' is the annual interest rate.

The variance of energy usage cost based on annual payments for $T = \frac{\tau}{12}$ years is given as

$$Var[\pi_e(T)] = E[\pi_e(T)^2] - E[\pi_e(T)]^2.$$

By using $\pi_e(T)$ given in Equation (8), the above equation can be expressed as

$$Var[\pi_e(T)] = E\left[\sum_{t=0}^T \sum_{t'=0}^T (1+r')^{-t} G^2 (12mt + n + X_t) (1+r')^{-t'} (12mt' + n + X_{t'})\right] - E[\pi_e(T)]^2.$$

where r' is the annual interest rate. Finally, the variance of energy usage cost is given as

$$Var[\pi_e(T)] = \frac{G^2 \sigma^2 B(T)}{(1-\varphi^2)} + \frac{2G^2 \sigma^2}{(1-\varphi^2)} \sum_{t=0}^{T-1} (1+r')^{-2t} \frac{\varphi - \varphi\left(\frac{\varphi}{(1+r')}\right)^{T-t}}{(1+r') - \varphi}$$

$$= \frac{G^2 \sigma^2 B(T)}{(1 - \varphi^2)} + \frac{2G^2 \sigma^2 \varphi}{(1 - \varphi^2)((1 + r) - \varphi)} \left(\frac{(1 + r')^{2-2T}((1 + r')^{2T} - 1)}{(1 + r')^2 - 1} - \left(\frac{\varphi}{(1 + r')} \right)^T \frac{((1 + r')\varphi)^{-T+1}(((1 + r')\varphi)^T - 1)}{((1 + r')\varphi) - 1} \right),$$

where $B(T) = \frac{(1+r')^{-2T}((1+r')^{2(T+1)}-1)}{(1+r')^2-1}$.

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