Einstein-Podolsky-Rosen type quantum entanglement between coupled cavities

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Abstract. We investigate Einstein-Podolsky-Rosen (EPR) type spatial entanglement between two coupled, driven, dissipative, nonlinear optical cavities. We identify required parameter regimes of polariton-exchange and nonlinearity coefficients to have robust EPR type entanglement at the steady state. In addition, we examine the influence of weak and strong drives on these parameter regimes.

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1. Introduction

Coupled nonlinear cavity arrays are considered to be ideal simulators of quantum many body phenomena [1]. Their building block is a simple two site system as shown in Figure 1. Non-local interaction is provided by the photon tunneling between the cavities while the material inside the cavities hybridize photons into polaritons subject to nonlinear local (on-site) interaction. An earlier study of such a system

Figure 1. (Color Online) Two site system of coupled nonlinear cavities. Overlap of the cavity fields (solid curves) yields exchange of the photons between the cavity sites. Cavities contain atoms or quantum dots (not shown) which hybridized with the photons to form polaritons subject to nonlinear on-site self interaction.

of two coupled cavities **with Kerr-type nonlinearity** [2] revealed a curious interplay between coherence and localization of the polaritons. It is concluded that polaritons are coherent and delocalized over the cavities when the tunneling exchange is stronger than the nonlinearity. In the opposite case of weaker tunneling, polaritons are localized in each cavity and antibunched [3].

In the present work, we explore Einstein-Podolsky-Rosen (EPR) [4] type spatial quantum correlations between the sites in this system. EPR type entanglement is a key resource in quantum communication for its use in quantum key distribution, teleportation, entanglement swapping and dense coding [5]. Determination of realistic parameter regimes for EPR type entanglement in coupled cavity arrays would bring such systems closer for practical quantum network operations. In particular it is crucial to ensure the entanglement is robust under open system conditions. **This robustness is the main difference of these systems from the quantum many body systems they are simulating.** Accordingly, we take into account the influence of drive and dissipation and examine the quantum correlations in the steady state.

Our specific objective is to determine the required parameter regimes of the polariton tunneling rate and nonlinearity coefficient, under drive and dissipation conditions, to realize robust EPR type spatial entanglement of the cavities at the steady state. For that aim we numerically solve the master equation describing the dynamics of the driven-dissipative open system to find the steady state density matrix, which is subsequently used to evaluate steady state behavior of a measure of EPR type entanglement [6]. By repeating this calculation process for ranges of tunneling rate and nonlinearity parameter relative the dissipation under weak and strong drive conditions we characterize a parametric phase diagram showing a region of EPR type correlations survived in the steady state.

2. The Model System

We consider a system of two identical nonlinear cavities labeled with $i = 1, 2$, coupled by single photon exchange interaction, as shown in Figure 1. We assume the cavities contain atoms or quantum dots whose quanta is hybridized with the photons and become polaritons. Both cavities are driven by a coherent pump at rate F at the laser frequency ω_L . The corresponding model Hamiltonian in a frame rotating at ω_L can be written as [2]

$$\hat{H} = \sum_{i=1,2} (U \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i \hat{b}_i + F \hat{b}_i^\dagger + F^* \hat{b}_i) + J(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1), \quad (1)$$

where U is the nonlinearity parameter and J is the polariton tunneling coefficient. Annihilation (creation) operator for the polaritons with frequency ω_i is denoted by $\hat{b}_i$ ($\hat{b}_i^\dagger$). We take $\omega_i = \omega_L$ for which the free energy terms vanish.

This model without the drive term is the two-site building block for the extended system of coupled cavity arrays [1]. It is the quantum simulator for the two-site Kerr-Hubbard or Bose-Hubbard models that may be realized in settings other than the coupled cavities. In particular the model, without the drive term, is known as the Lipkin-Meshkov-Glick model in nuclear physics [7]; while it is called as two-mode Bose-Einstein condensate (BEC) model in the physics of ultracold atoms [8–13].

EPR type entanglement in the context of BECs trapped in a double-well potential has been discussed recently [14–16]. Earlier quantum optical settings of generating EPR entanglement between propagating bright laser fields utilize sequential application of Kerr nonlinearity in crystals and mixing of fields at beam splitters [5]. Atomic models reveals that simultaneous nonlinearity and mixing interactions can also be used for EPR correlations instead of sequential optical procedure. In both cases, Kerr type nonlinearity creates squeezing of the quadrature noise of the relevant continuous variables of the system locally, while the linear mixing interaction redistributes the noise for nonlocality such that within certain parameter regimes the EPR correlation condition can be satisfied. Our examination generalize the quantum optical method from propagating fields to localized cavity fields and takes into account the open system conditions as well, in contrast to atomic and optical methods.

3. Results and Discussions

To investigate the quantum dynamics of our model systems under dissipation, we assume that the coupling of the cavity photons and the reservoir photons is weak; and the correlation time of the reservoir photons is negligibly short. Under these so called Born and Markov conditions, the dynamics of the open system can be determined by solving the master equation

$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] + \sum_{i=1,2} \kappa_i \mathcal{D}[x] \hat{\rho}. \quad (2)$$

Here, κ_i are the photon loss rates out of the cavities. $\mathcal{D}[x]\hat{\rho} = [2\hat{x}\hat{\rho}\hat{x}' - \hat{x}'\hat{x}\hat{\rho} - \hat{\rho}\hat{x}'\hat{x}]/2$ are the Liouvillian superoperators in the Lindblad form. We assume $\kappa_i \equiv \kappa$ and coherent pump amplitude F is taken to be real.

We solve the master equation for the steady state density operator using the QuTiP: Quantum Toolbox in Python software [17]. The master equation is solved by taking $N_1 = N_2 = 4$ for the Fock space dimensions of each cavity field. When we

Figure 2. (Color Online) Dependence of α (a) $F/\kappa = 0.1$, (b) $F/\kappa = 1$ with respect to dimensionless parameters J/κ and U/κ in two-site KH system with single-photon exchange. α is multiplied by 10^3 in (a) for visibility.

increase the Fock space dimension up to 8 and remake our calculations, we find that the results remain unchanged. We consider two cases distinguished by the action of weak $F/\kappa = 0.1$ and strong pump $F/\kappa = 1$. We take $\kappa/2\pi = 0.4$ MHz. The parameters we use in the simulations are within the ranges accessible in present circuit QED systems [18, 19].

Consider two EPR-like operators $\hat{u}$ and $\hat{v}$ defined by

$$\hat{u} = |c|\hat{x}_1 + \frac{1}{c}\hat{x}_2 \quad (3)$$

$$\hat{v} = |c|\hat{p}_1 - \frac{1}{c}\hat{p}_2 \quad (4)$$

where dimensionless canonical continuous variables for our system are taken to be $\hat{x}_i = (\hat{b}_i + \hat{b}_i^\dagger)/\sqrt{2}$ and $\hat{p}_i = (\hat{b}_i - \hat{b}_i^\dagger)/i\sqrt{2}$ with $i = 1, 2$. They obey the canonical commutation rule $[x, p] = i$. Presently c is an arbitrary nonzero parameter to be determined [20] for EPR entanglement condition given by the inequality [6]:

$$(\Delta\hat{u})^2 + (\Delta\hat{v})^2 < c^2 + \frac{1}{c^2}. \quad (5)$$

If we choose $c^2 = \sqrt{\langle\hat{n}_2\rangle/\langle\hat{n}_1\rangle}$, then the entanglement criteria can be written in terms of a parameter α defined as [20]:

$$\alpha = c^2\langle\hat{n}_1\rangle + \frac{1}{c^2}\langle\hat{n}_2\rangle + \frac{c}{|c|}\langle\hat{b}_1^\dagger\hat{b}_2^\dagger + \hat{b}_1\hat{b}_2\rangle \quad (6)$$

The negative values of this parameter indicates EPR-correlations between cavity fields. We run our simulations for the parameter regimes $0 \leq (J/\kappa, U/\kappa) \leq 10$, though we present only the relevant part of our results in Figure 2. In Figure 2(a), we plot the α parameter with respect to dimensionless parameters J/κ and U/κ for the case of $F/\kappa = 0.1$. In the regimes where $0 \leq U/\kappa \leq 10$ and $J/\kappa < 0.5$ quantum correlations are demolished. Similar results are obtained for $0 \leq J/\kappa \leq 10$ with weak nonlinearity. We see that quantum correlations start to occur for $0.5 \sim J/\kappa$ with stronger nonlinear interactions, i.e., for every point where there is a correlation between field modes we have $U/J > 1$.

In Figure 2(b), we plot the α parameter with respect to dimensionless parameters J/κ and U/κ for the case of $F/\kappa = 1$. The minimum value of tunneling J/κ to see EPR correlations between the field modes is shifted from ~ 0.5 to ~ 1.5 in such a way that the relation $U/J > 1$ preserved and hence the region corresponds to the EPR correlated modes is narrowed. An optimal value of the drive strength is needed to compensate the cavity loss while balancing its harmful effect on the EPR correlations. The drive field brings coherence on the system while one needs squeezing of the polariton quadratures essentially for EPR type entanglement.

In both figures the profound quantum character of the system can only be found once the tunneling rate and the nonlinearity beats the decoherence induced by the cavity loss. We find about 5 times stronger nonlinear interaction coefficients than the cavity loss is required to enter the quantum regime. While the tunneling rate is more tolerant and it is sufficient to be just larger than the loss rate. On the other hand, there is another stringent constraint on the tunneling rate. While tunneling is needed for nonlocal correlations towards spatial EPR type entanglement, it should not be stronger than the local nonlinear interaction. We see that in both weak and strong drive cases, the linear tunneling rate should be about smaller than half of the nonlinear coefficient.

4. Conclusions

We explored EPR type entanglement between coupled cavities with Kerr nonlinearity under drive and dissipation in the steady state of the system. We find that if the nonlinear interaction is about five times stronger than the cavity loss, the tunneling rate is greater than the cavity loss but less than about half of the nonlinearity coefficient, EPR entanglement becomes robust and survive in the steady state. The condition on tunneling rate relative to nonlinearity depends on the drive strength. **In particular, when we increase the drive to $F/\kappa = 2.5$, EPR entanglement is lost within our parameter regime. Additionally, decreasing the drive to $F/\kappa = 0.025$ leads to lower tunneling rates to see EPR entanglement.** As the drive contributes towards coherence rather than squeezing, the tunneling rate should be reduced even more in the presence of strong drive. Our results contribute towards practical quantum communication and quantum networking with continuous variable entanglement using coupled cavity arrays.

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